On the Computations of Homological Holes
where topology, geometry and combinatorics meet

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PhD thesis supervised by Alexandra Bac, Aldo Gonzalez-Lorenzo

7th October 2026





Algebraic topology



Figure: Topology joke, by Henry Segerman.

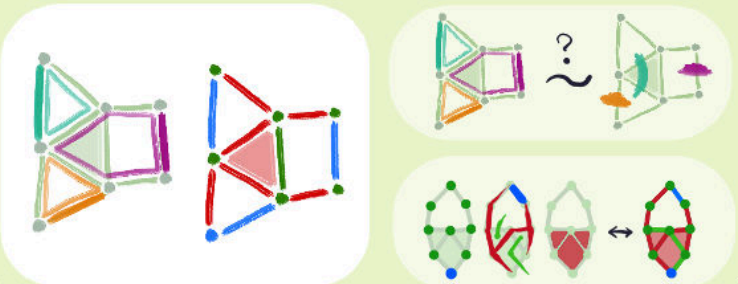
Subfields

Homotopy, **homology**, knot theory...

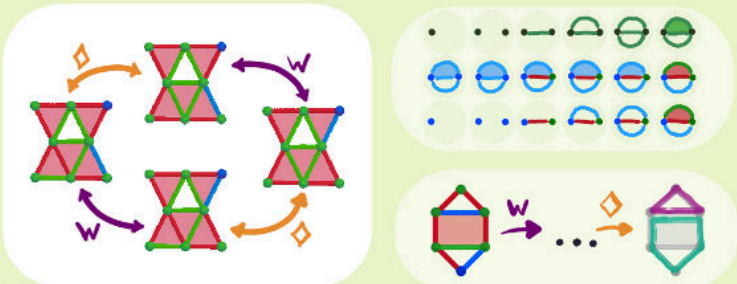
Outline of the presentation

Introduction to Homology

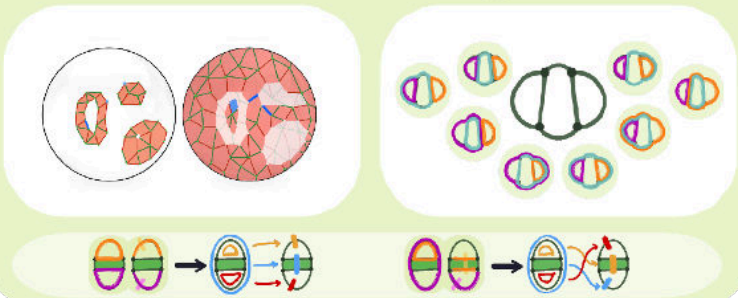
Contribution 1. Combinatorial Properties of HDVFs



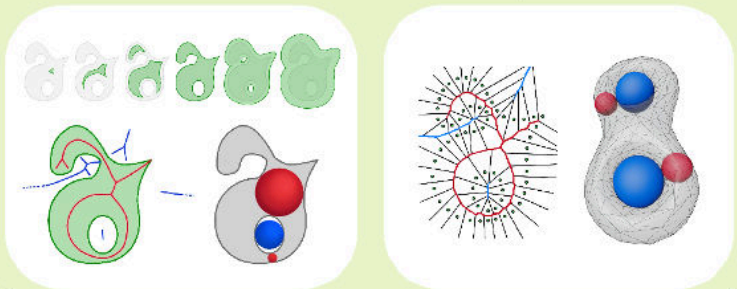
Contribution 2. The Space of HDVFs



Contribution 3. Beyond Standard Homology Information

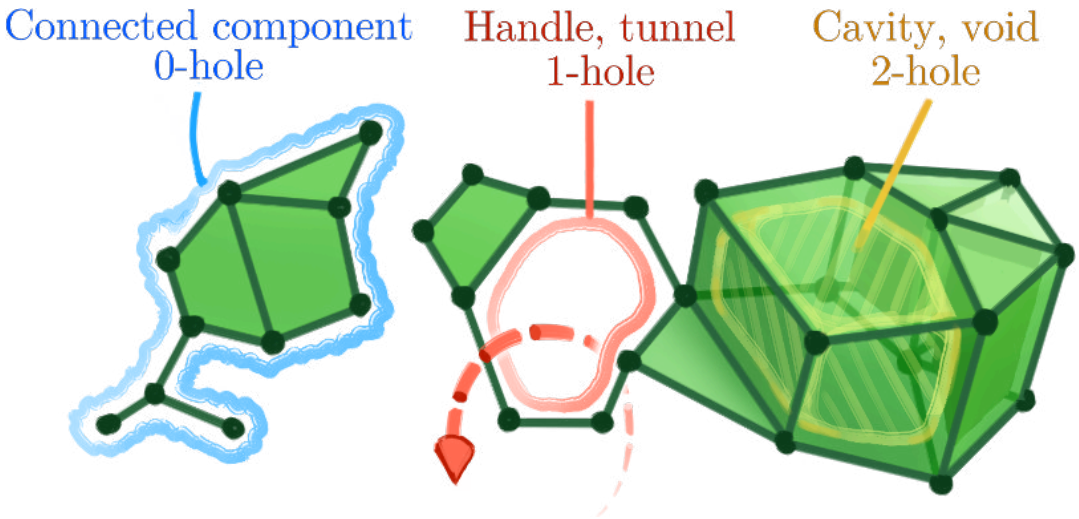


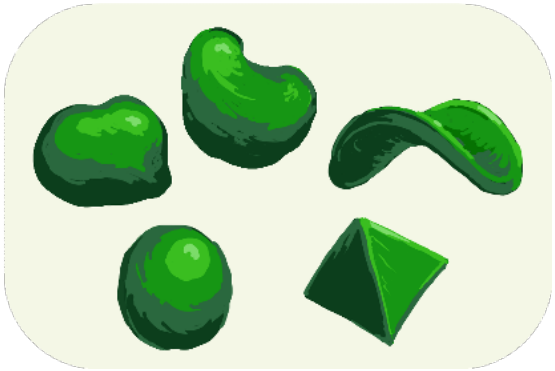
Contribution 4. Geometric Measures of Homological Holes



Conclusion and Prospects

Homology and Hole Intuition





0-holes: $\beta_0 = 5$



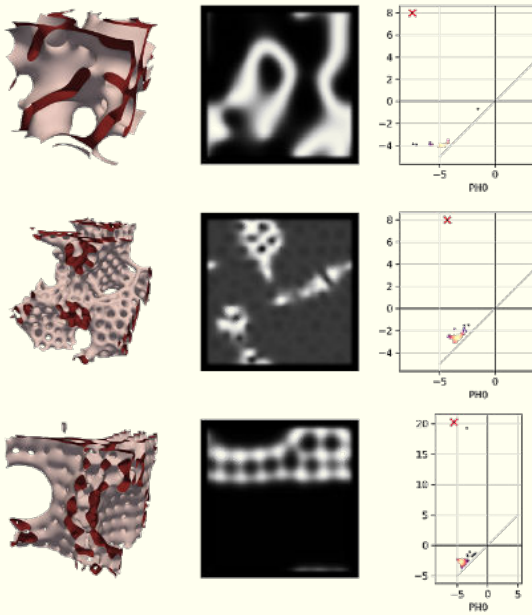
1-holes: $\beta_1 = 6$



2-holes: $\beta_2 = 2$

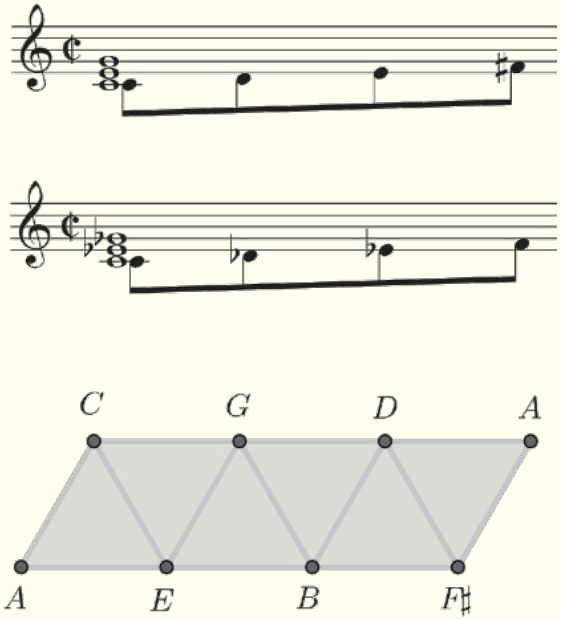
Some applications

Shape analysis



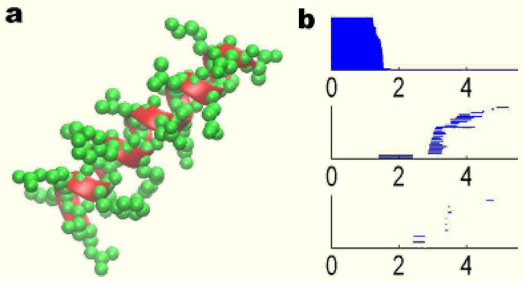
A.Song et al. *Generalized Morse Theory of Distance Functions to Surfaces for Persistent Homology* (2025).

Music analysis



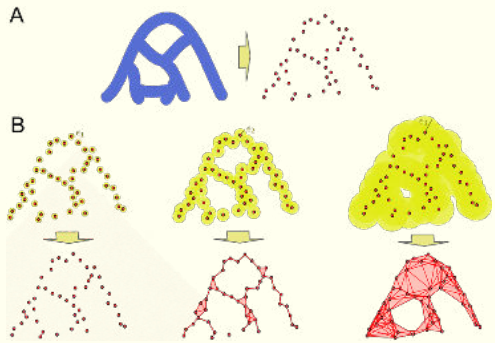
M.Bergomi and A.Baraté. *Homological persistence in time series: an application to music classification* (2020).

Protein analysis



K.Xia and G-W.Wei. *Persistent homology analysis of protein structure, flexibility, and folding* (2014)

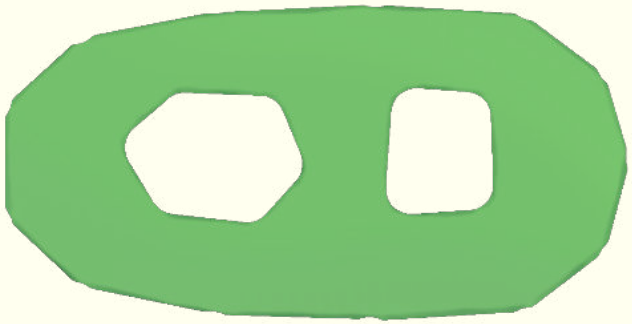
Topological data analysis



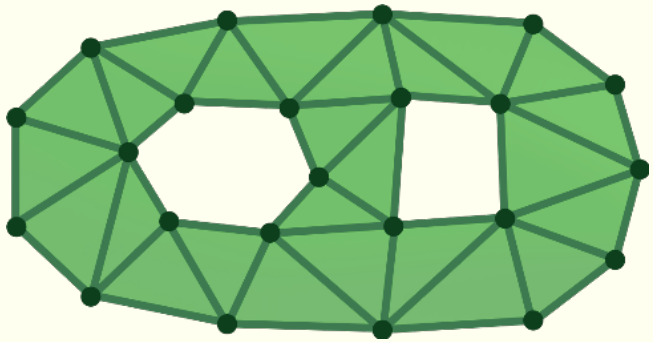
P.Camara et al. *Inference of Ancestral Recombination Graphs through Topological Data Analysis* (2015).

Geometric complexes / Chain complexes

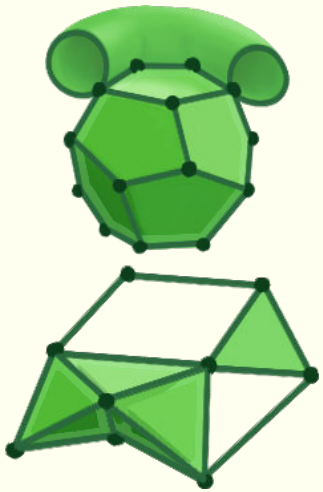
Continuous shape

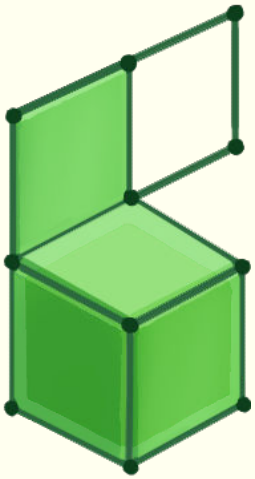


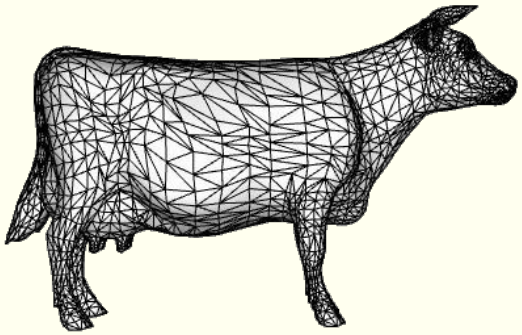
Discrete shape

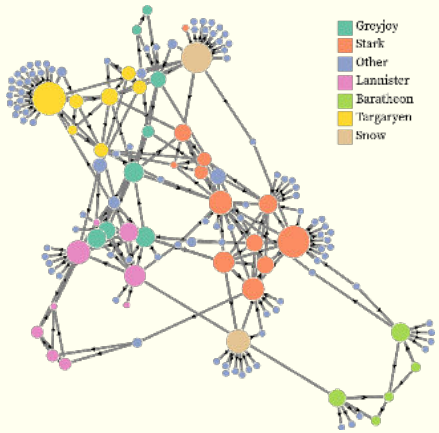


Discrete shape examples

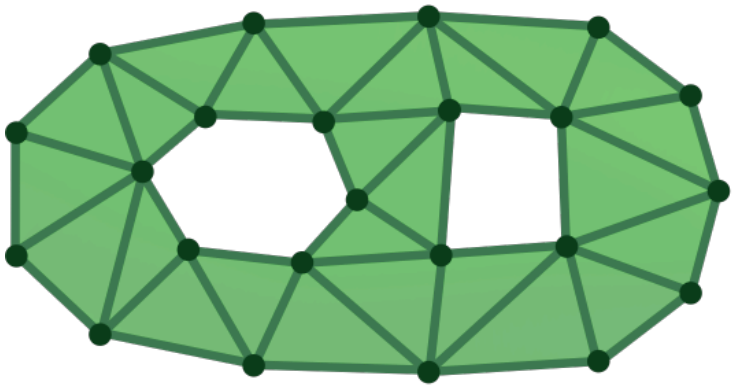








Cycles, boundaries, holes and homology

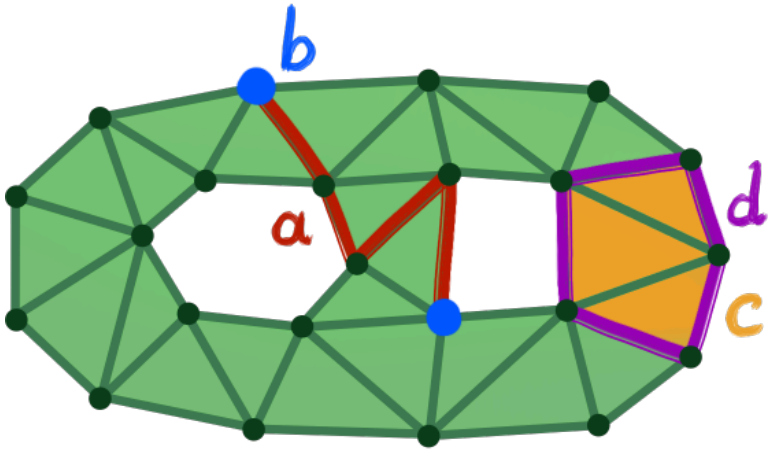


Chain complex

$$K_n \xrightarrow{\partial_n} \dots \xrightarrow{\partial_{q+1}} K_q \xrightarrow{\partial_q} \dots \xrightarrow{\partial_1} K_0 \xrightarrow{\partial_0} 0$$

- K_q: the space of q-**chains**.
- $\partial_q : K_q \to K_{q-1}$: the q-**boundary** map.
- $\partial_q \circ \partial_{q+1} = 0$.

Cycles, boundaries, holes and homology



Chain complex

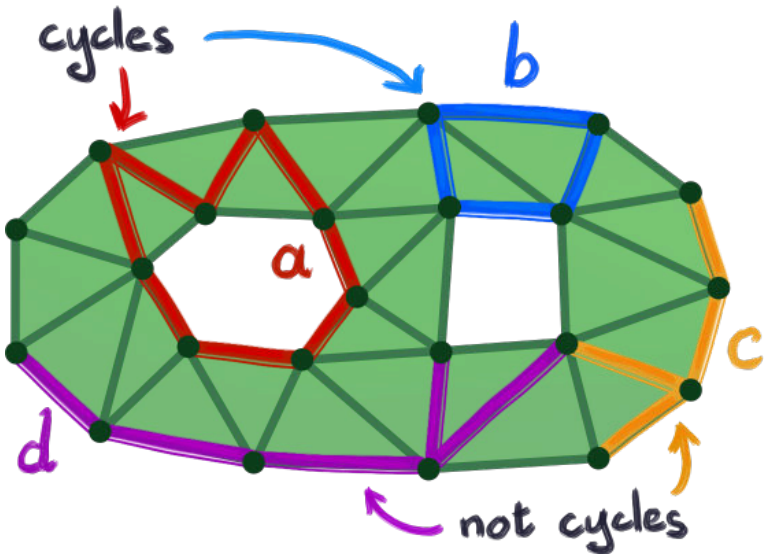
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- K_q: the space of q-**chains**.
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chain: $\mathbf{a} = \lambda_1 \cdot \tau_1 + \dots + \lambda_m \cdot \tau_m$

$\mathbf{b} = \partial_1(\mathbf{a})$ $\quad$ $\mathbf{d} = \partial_2(\mathbf{c})$

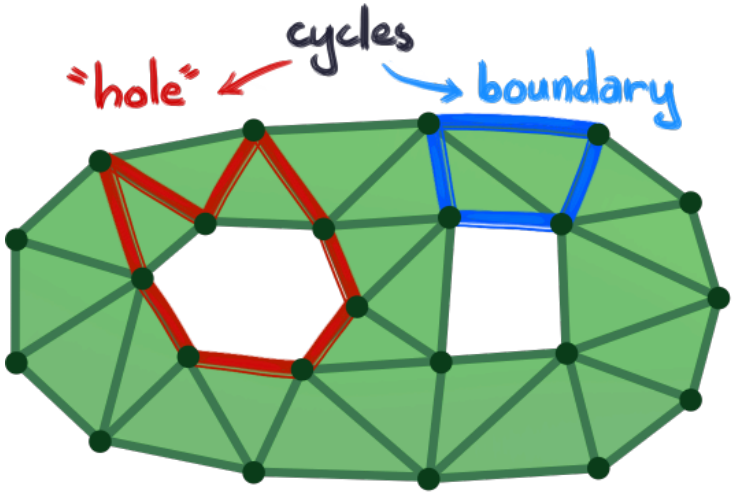
Cycles, boundaries, holes and homology



Definition. Cycle

A **cycle** is a chain that does not have any boundary (e.g. loops).

Cycles, boundaries, holes and homology



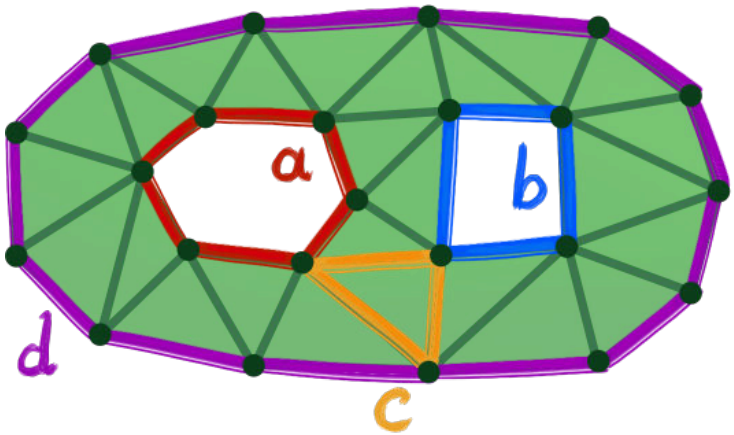
Definition. Cycle

A **cycle** is a chain that does not have any boundary (e.g. loops).

The **boundary** of a chain is a **cycle** ($\partial_q \circ \partial_{q+1} = 0$).

A cycle either is a **boundary**, either **surrounds a hole**.

Cycles, boundaries, holes and homology



$H_1(K)$ has **4** elements

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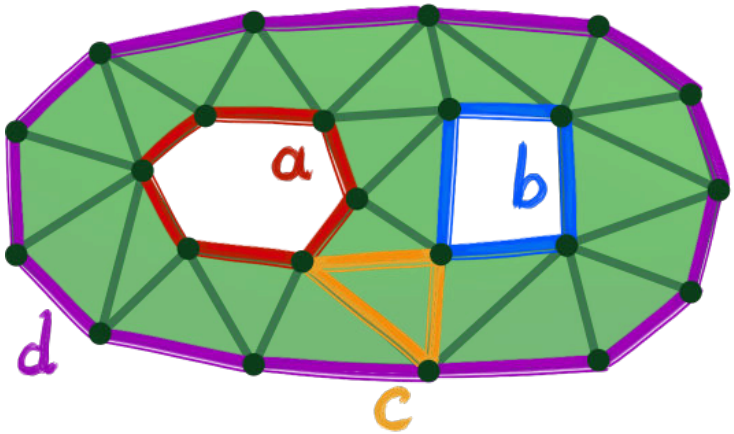
A cycle either is a **boundary**, either **surrounds a hole**.

Definition. Homology group

The q-**homology group** of a complex K classifies how q-cycles can surround holes (or not).

$$H_q(K) = \frac{\ker \partial_q}{\text{im } \partial_{q+1}}$$

Homology information



Theorem. Homology group structure

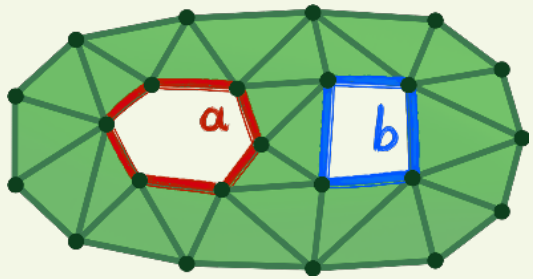
$$H_q(K) \approx \mathcal{R}^{\beta_q} \oplus T$$

with $\mathcal{R}$ the ring, and T the **torsion**.
β_q is the q-th **Betti number**.

$H_1(K) \approx (\mathbb{Z}/2\mathbb{Z})^2$ $\quad$ $\beta_1 = 2$

Definition. Homology basis

A q-**homology basis** is a set of β_q cycles that **generates** every class of $H_q(K)$.

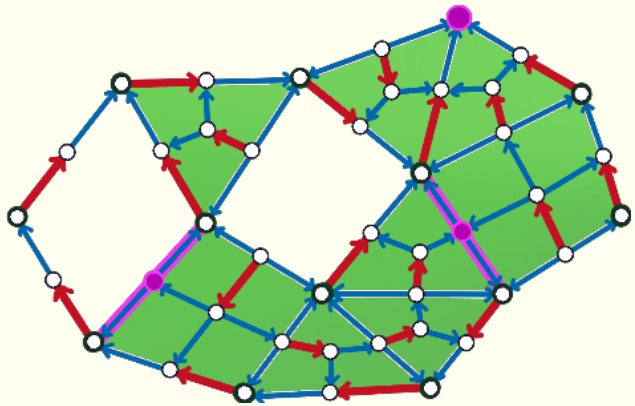


How to compute homology?

Smith normal form

$$\partial = P \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & \ddots & 0 & \cdots & 0 \\ 0 & 0 & 1 & & 0 \\ \vdots & & & 0 & \vdots \\ 0 & & \cdots & & 0 \end{pmatrix} Q$$

Discrete Morse theory (R.Forman 2001)



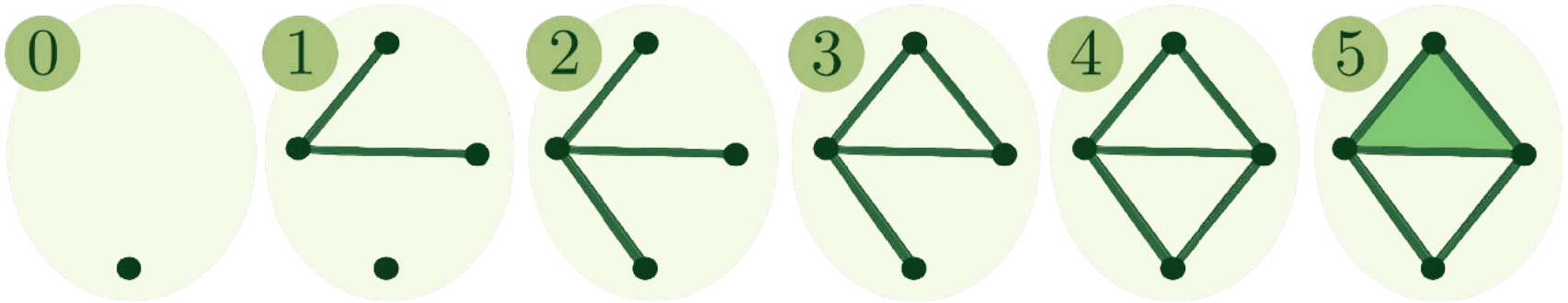
Effective homology (F.Sergeraert 1986)

$$\begin{array}{ccccccccc} \cdots & \underset{h_q}{\overset{\partial_{q+1}}{\rightleftarrows}} & K_q & \underset{h_{q-1}}{\overset{\partial_q}{\rightleftarrows}} & K_{q-1} & \underset{h_{q-2}}{\overset{\partial_{q-1}}{\rightleftarrows}} & \cdots \\ & & g_q \uparrow \downarrow f_q & & g_{q-1} \uparrow \downarrow f_{q-1} & & \\ \cdots & \xrightarrow{d_{q+1}} & K'_q & \xrightarrow{d_q} & K'_{q-1} & \xrightarrow{d_{q-1}} & \cdots \end{array}$$

Persistent Homology

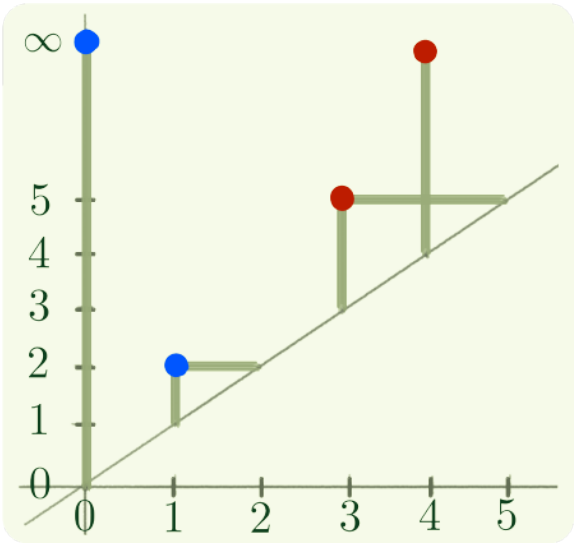
Definition. Filtration

A **filtration** is a growing sequence of complexes.



Persistent homology

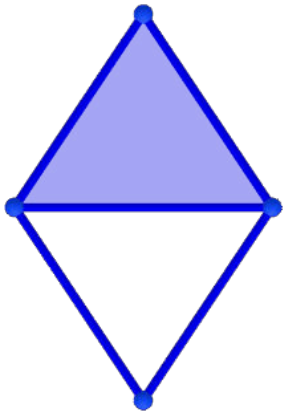
Tracks the **birth** and **death** of holes along a **filtration**. It encodes this information in a **persistence diagram**.

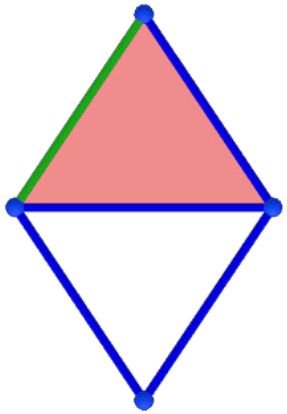


HDVF definition

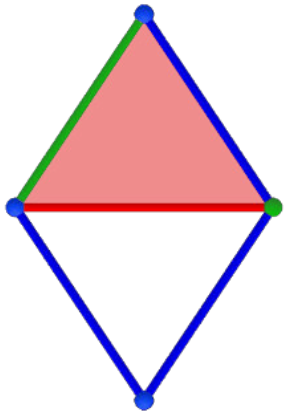
Definition. HDVF (A.Gonzalez-Lorenzo et al. 2016)

A **Homological Discrete Vector Field** (HDVF) is a partition of the cells in three categories, primary, secondary and critical cells, such that $\partial|^{\text{primary}}_{\text{secondary}}$ is **invertible**.

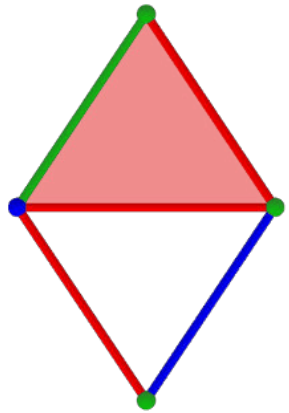




$$(1)$$



$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Reductions

Proposition. (A.Gonzalez-Lorenzo et al. 2016)

A HDVF (P, S, C) induces a **reduction** from (K, ∂) to (C, d).

Definition. Reduction (F.Sergeraert 1986)

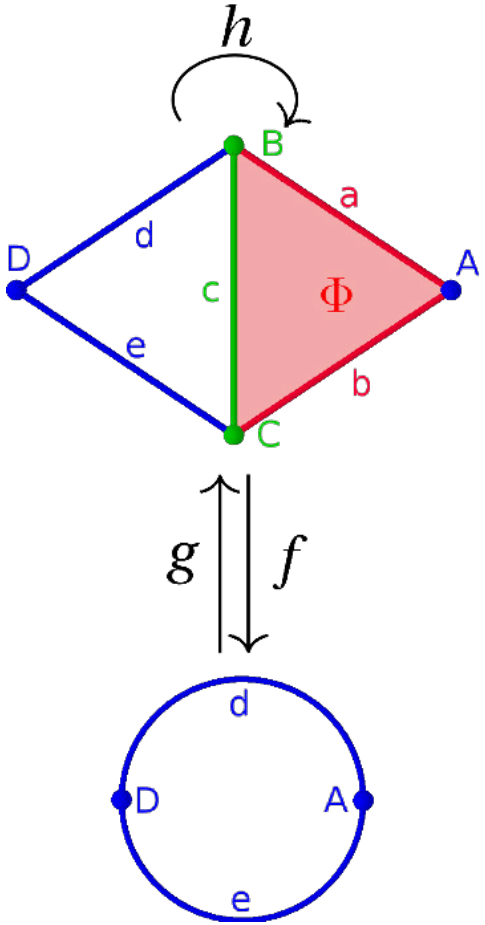
A **reduction** from a complex (K, ∂) to a complex (K', d) is a triplet of maps (f, g, h) satisfying some properties (not listed here).

$$\cdots \underset{h_q}{\overset{\partial_{q+1}}{\rightleftarrows}} K_q \underset{h_{q-1}}{\overset{\partial_q}{\rightleftarrows}} K_{q-1} \underset{h_{q-2}}{\overset{\partial_{q-1}}{\rightleftarrows}} \cdots$$

$$g_q \uparrow \downarrow f_q \qquad g_{q-1} \uparrow \downarrow f_{q-1}$$

$$\cdots \xrightarrow{d_{q+1}} K'_q \xrightarrow{d_q} K'_{q-1} \xrightarrow{d_{q-1}} \cdots$$

A reduction ensures that both complexes have **the same homology**.

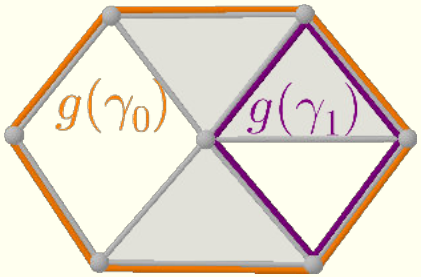


Perfect HDVFs

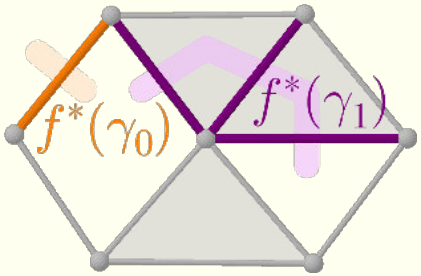
Definition. (A.Gonzalez-Lorenzo et al. 2016)

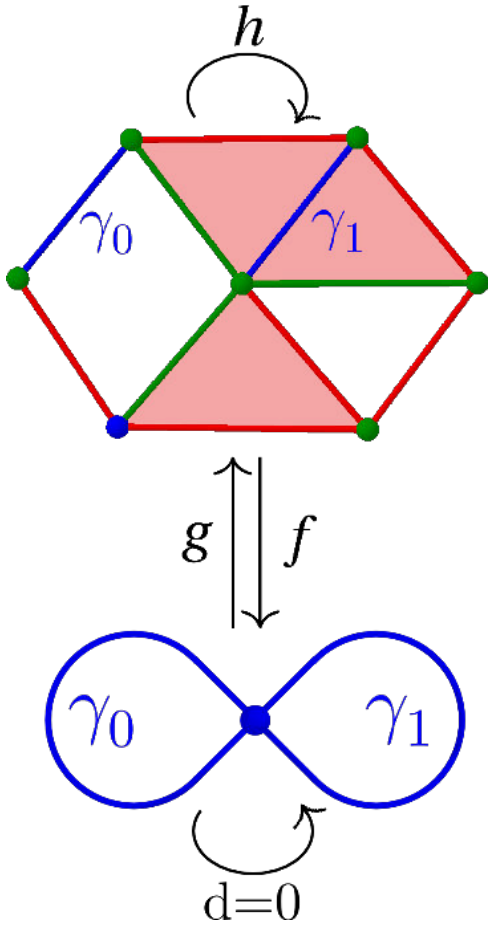
A HDVF and its reduction are **perfect** when the reduced boundary d **is zero**.

Induced homology basis



Induced cohomology basis

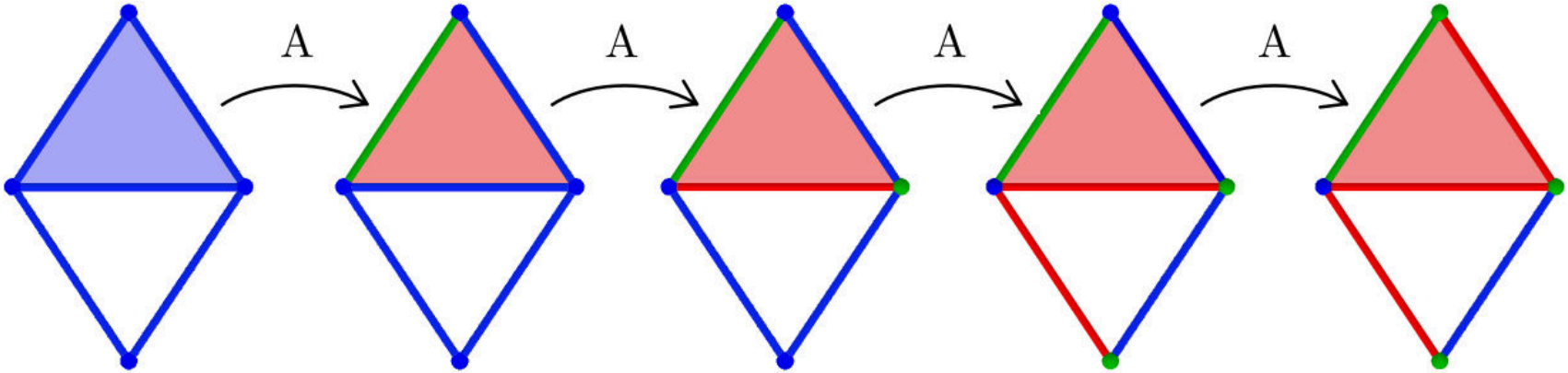




Completion and perfect HDVFs

Theorem. HDVF greedy completion (A.Gonzalez-Lorenzo et al. 2017)

A HDVF can always be **completed to a perfect HDVF**, by **greedily** adding primary-secondary pairs while maintaining the **HDVF condition**.[1]



"empty" HDVF

perfect HDVF

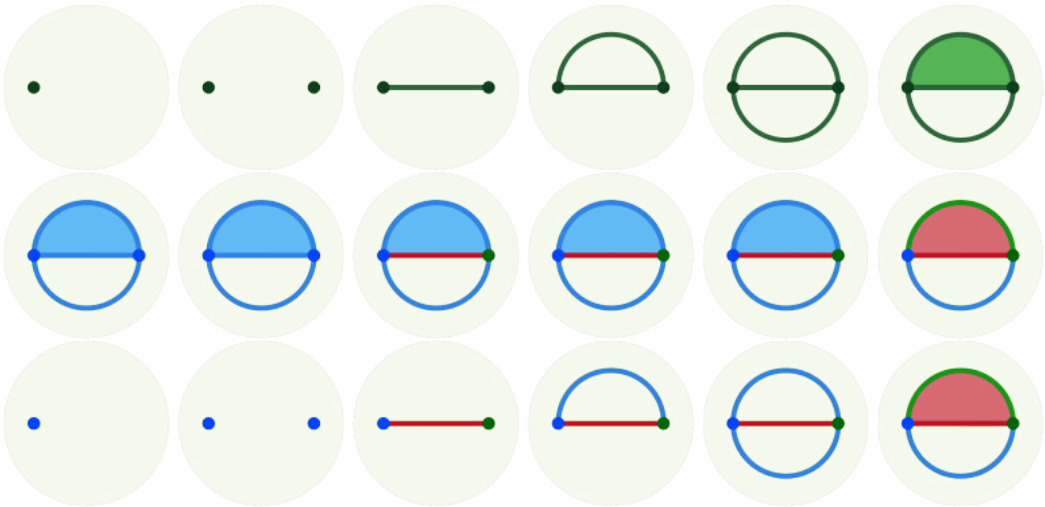
[1] This is only valid for homology over a field.

Encoding standard homology methods with HDVFs

Proposition. (A.Gonzalez-Lorenzo et al. 2017)

HDVF completions can encode:

- Discrete Morse theory computations.
- Standard persistent homology algorithms.
- Smith normal form computations.

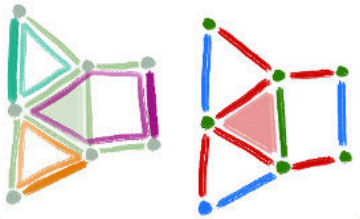


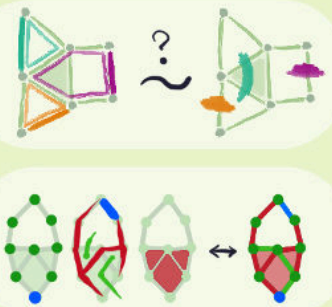
Persistence emulated by HDVF completion.

Contributions

Contribution 1. Combinatorial Properties of HDVFs

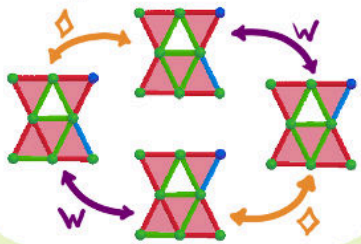
Characterization of the computed bases

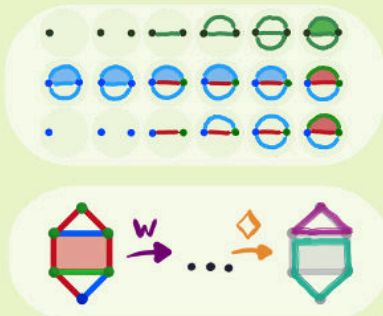




Contribution 2. The Space of HDVFs

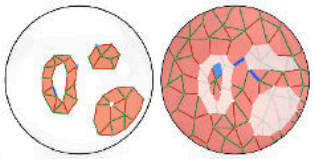
Connectivity of perfect HDVFs



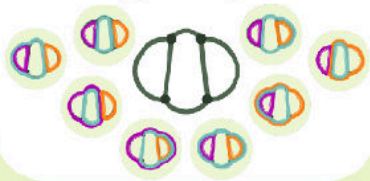


Contribution 3. Beyond Standard Homology Information

Constructive Alexander duality



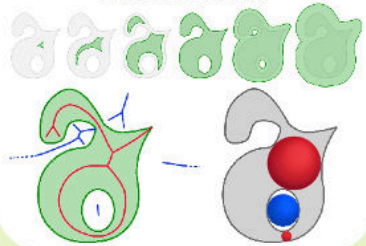
Homology configurations



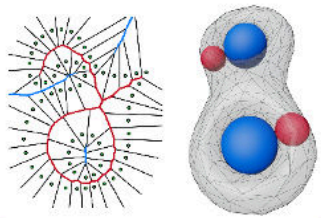


Contribution 4. Geometric Measures of Homological Holes

Hole balls from medial axes



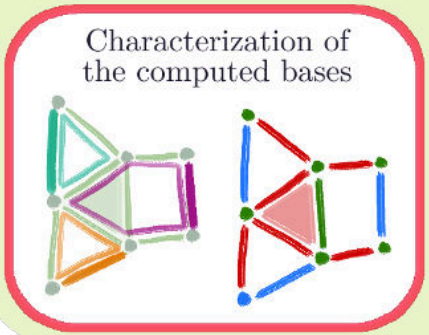
Discretization and implementation



Contributions

Contribution 1. Combinatorial Properties of HDVFs

Characterization of the computed bases

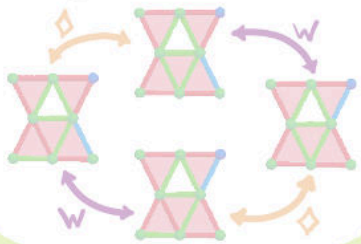


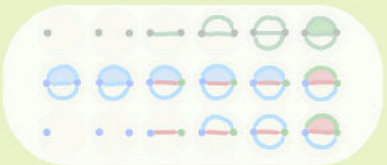




Contribution 2. The Space of HDVFs

Connectivity of perfect HDVFs

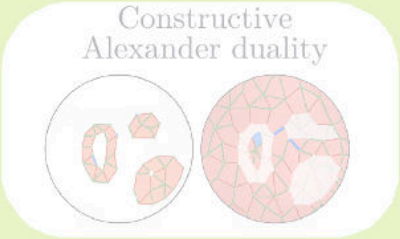




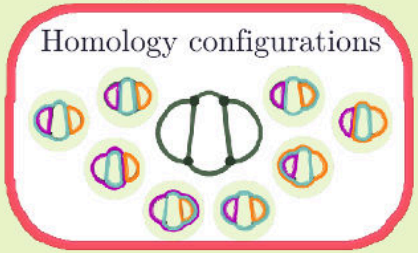


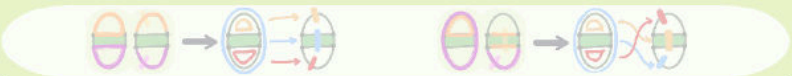
Contribution 3. Beyond Standard Homology Information

Constructive Alexander duality



Homology configurations



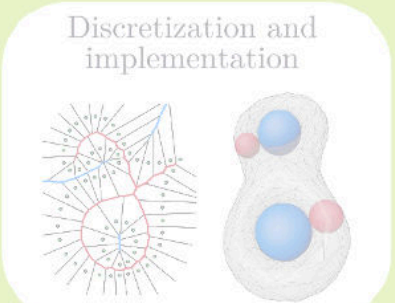


Contribution 4. Geometric Measures of Homological Holes

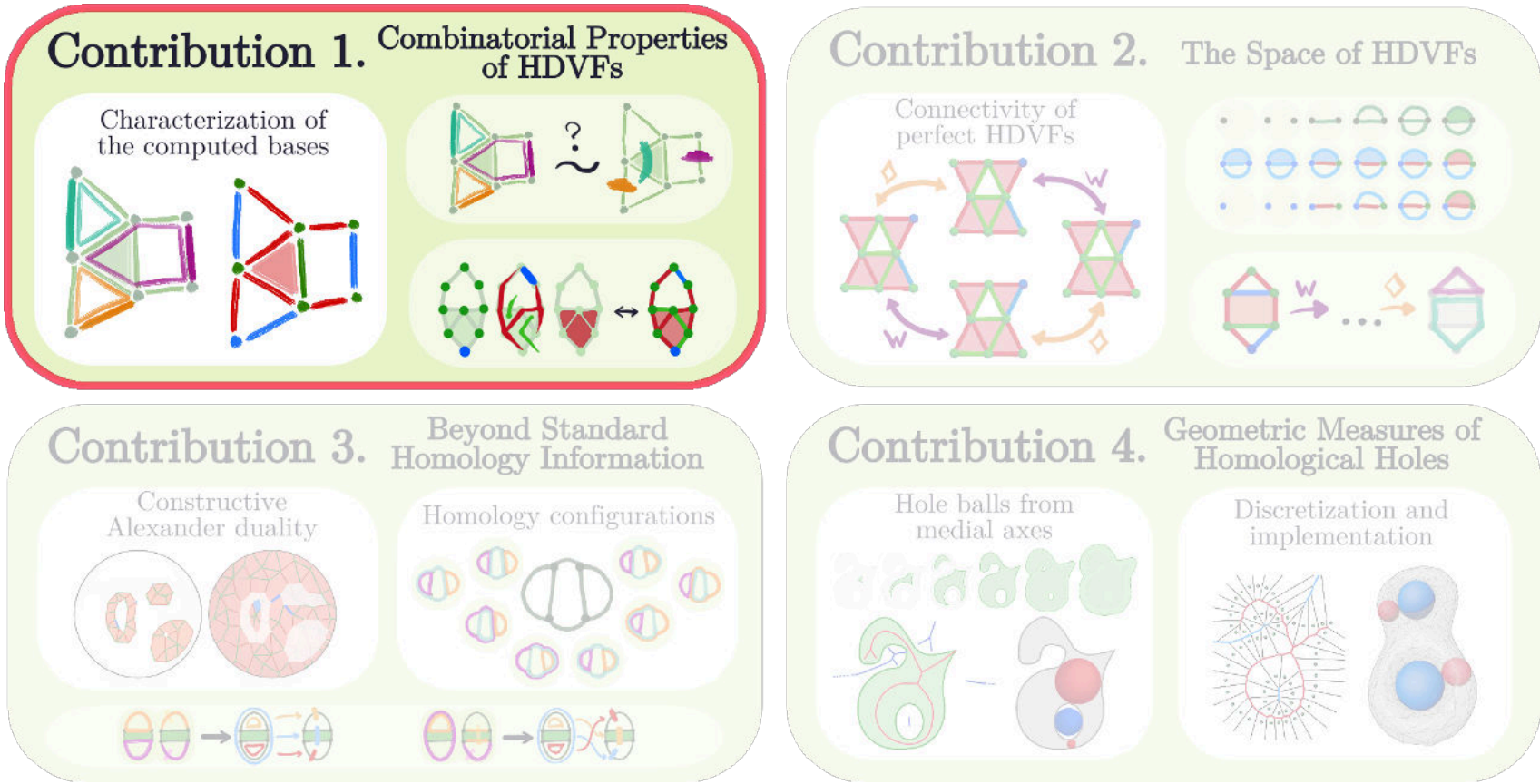
Hole balls from medial axes



Discretization and implementation



1. Combinatorial Properties of HDVFs



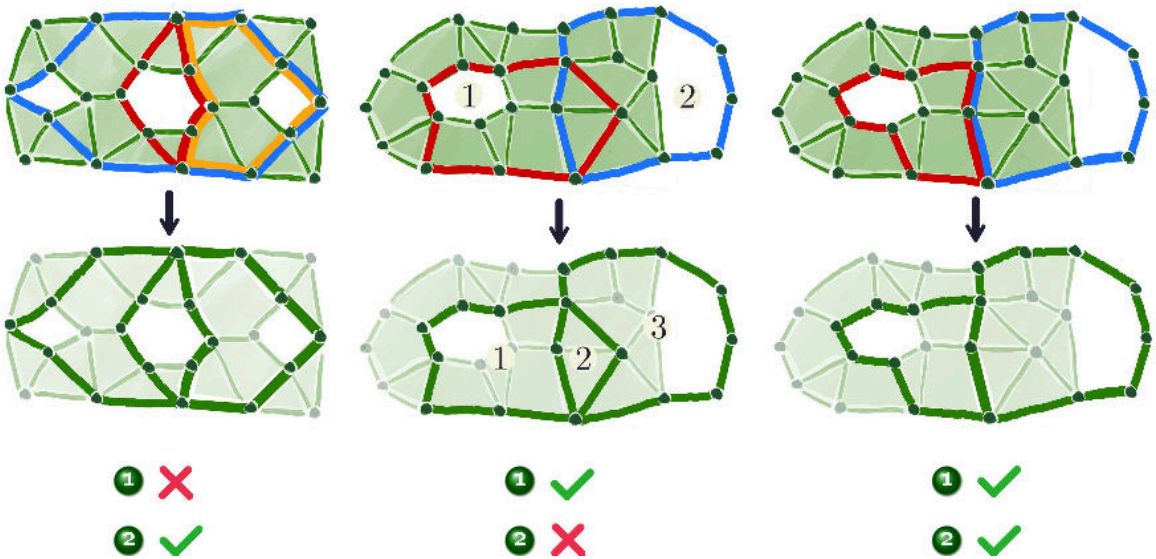
- **Yann-Situ Gazull**, Aldo Gonzalez-Lorenzo and Alexandra Bac. "Characterization of the Computed Homology and Cohomology bases". In *International Conference on Discrete Geometry and Mathematical Morphology* (Nov. 2025).
- **Yann-Situ Gazull**, Aldo Gonzalez-Lorenzo and Alexandra Bac. "Characterization of Homology and Cohomology bases Computed by Standard Methods". In: DGMM25 special issue, *Journal of Mathematical Imaging and Vision* (2026), **to be published**.

Characterization of the computed bases

Definition. Explicit homology basis

A q-homology basis is **explicit** if:

1. No generator is included in the union of the other generators.
2. The union of all generators has as many q-holes as the initial shape.



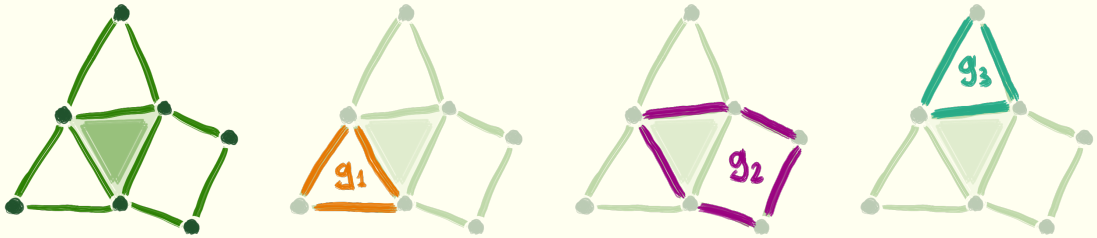
Theorem. (Y-S.Gazull et al. 2025)

A basis is **explicit** if and only if it is **induced by a perfect HDVF**.

Characterization of the computed bases

Being **explicit** $\Longrightarrow$ being **induced by a perfect HDVF**.

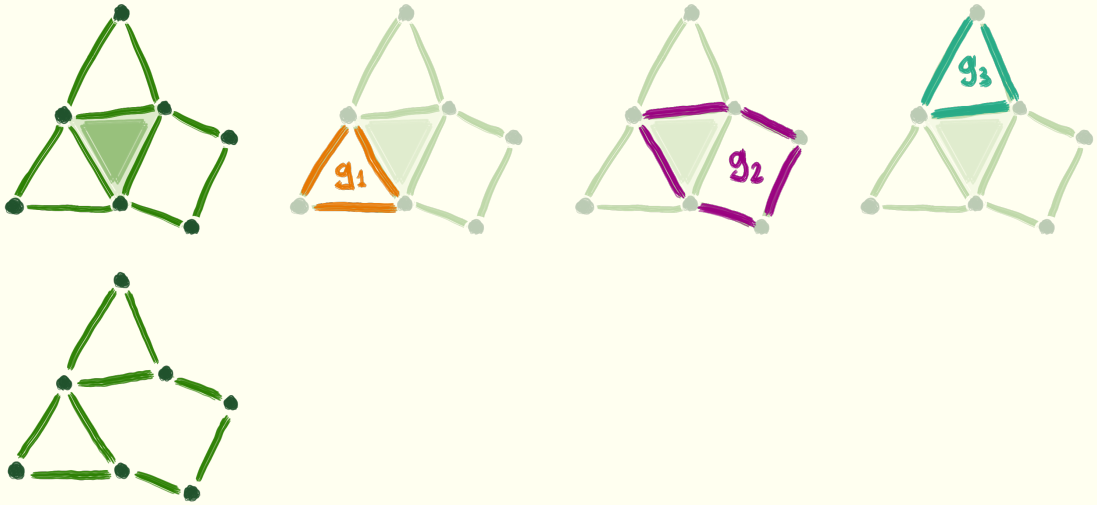
Proof.



Characterization of the computed bases

Being **explicit** $\Longrightarrow$ being **induced by a perfect HDVF**.

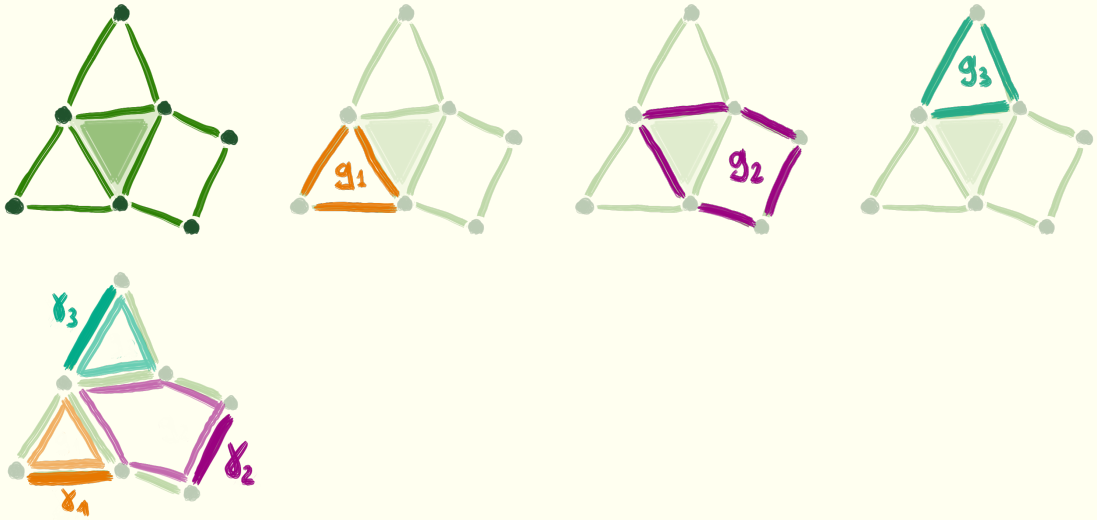
Proof.



Characterization of the computed bases

Being **explicit** $\Longrightarrow$ being **induced by a perfect HDVF**.

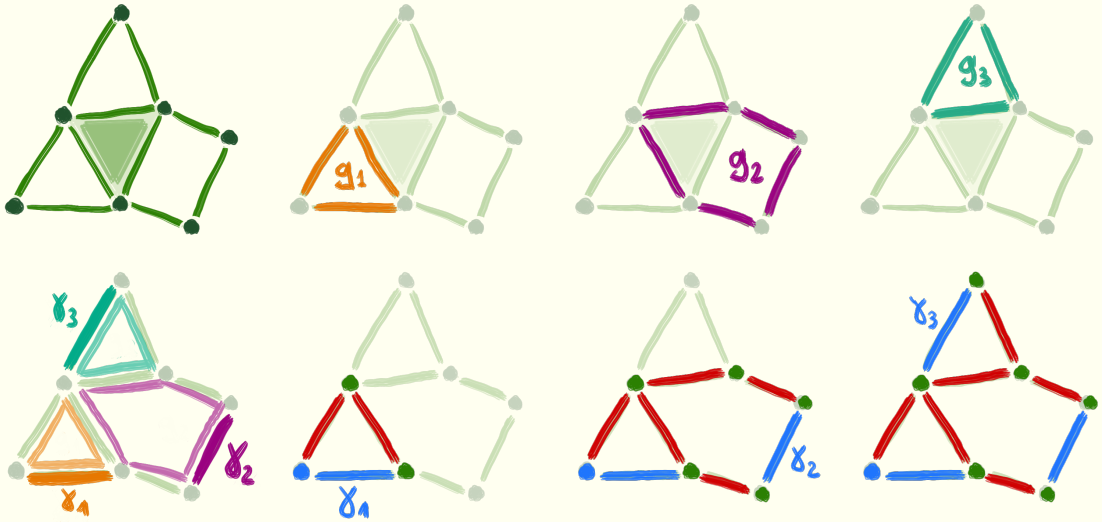
Proof.



Characterization of the computed bases

Being **explicit** $\Longrightarrow$ being **induced by a perfect HDVF**.

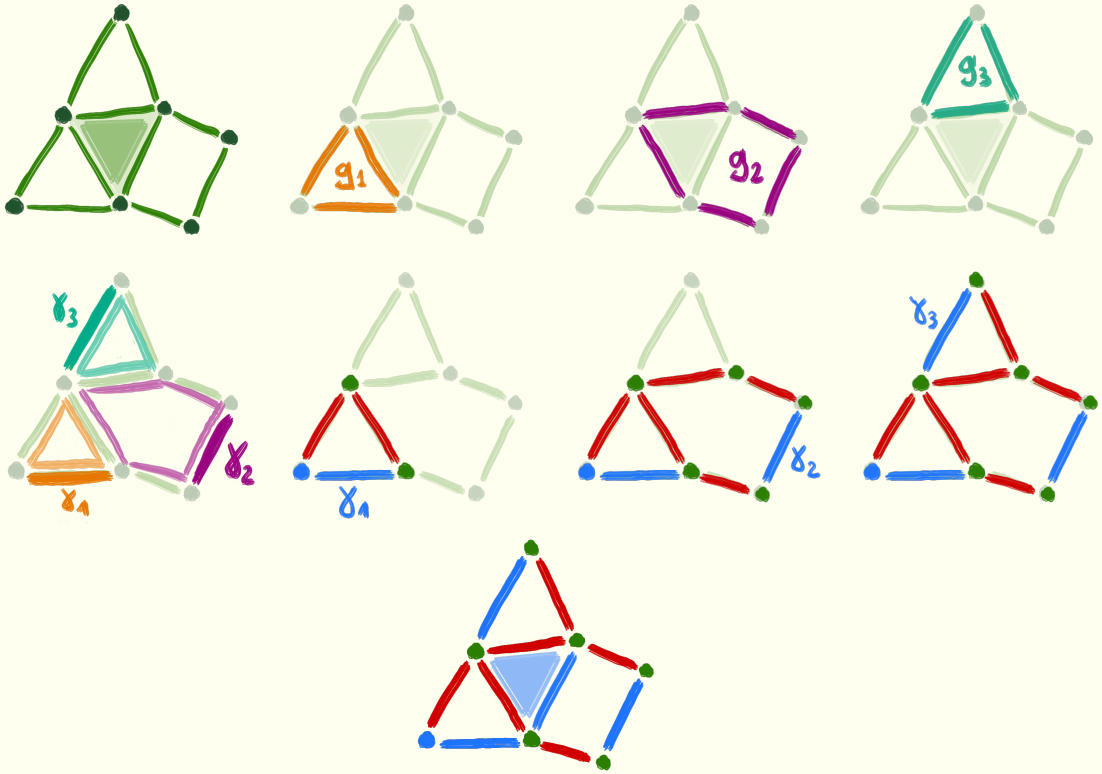
Proof.



Characterization of the computed bases

Being **explicit** $\Longrightarrow$ being **induced by a perfect HDVF**.

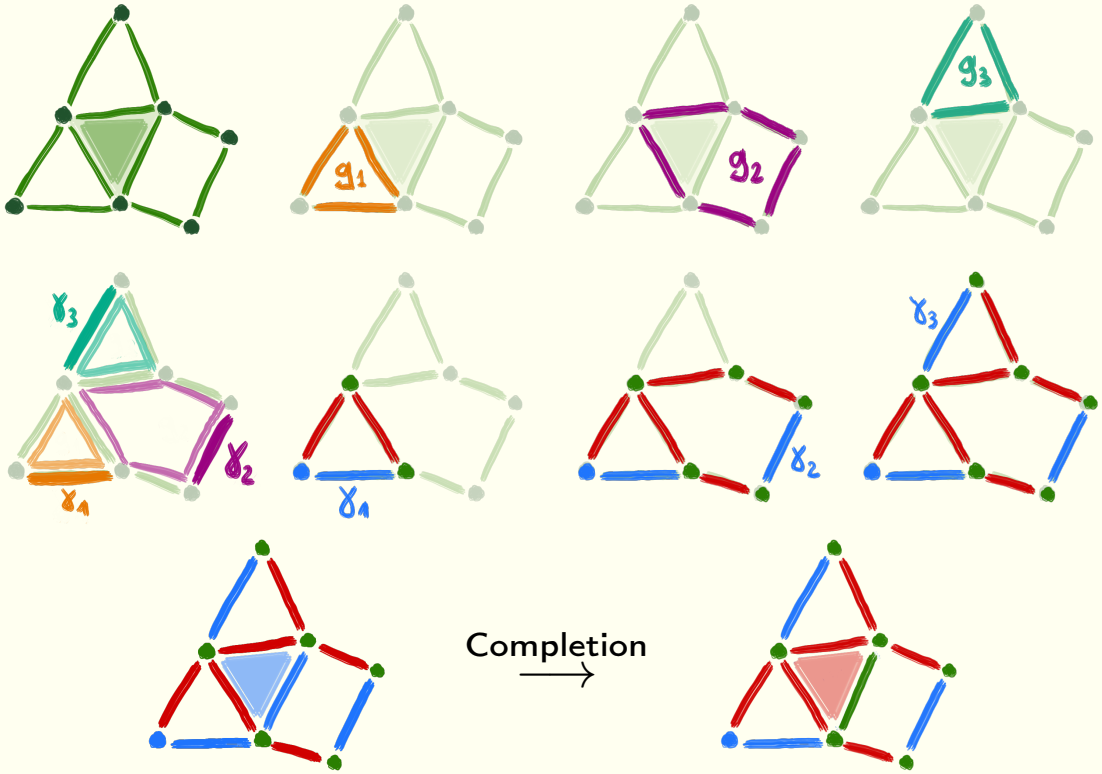
Proof.



Characterization of the computed bases

Being **explicit** $\Longrightarrow$ being **induced by a perfect HDVF**.

Proof.



Extensions of this characterization

Theorem. (Y-S.Gazull et al. 2025)

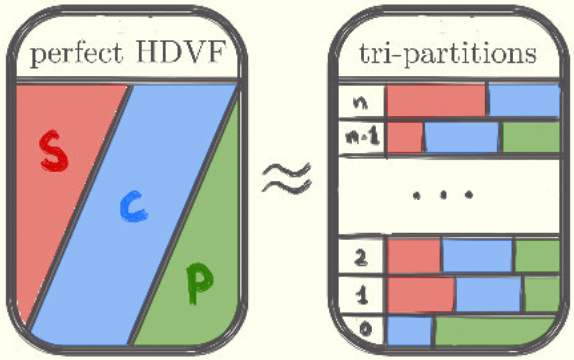
A basis is **explicit** if and only if it is **induced by a perfect HDVF**.

The result holds for **cohomology**.

We also characterize **the pairs of explicit homology and cohomology bases** that can be induced simultaneously **by a single HDVF**.

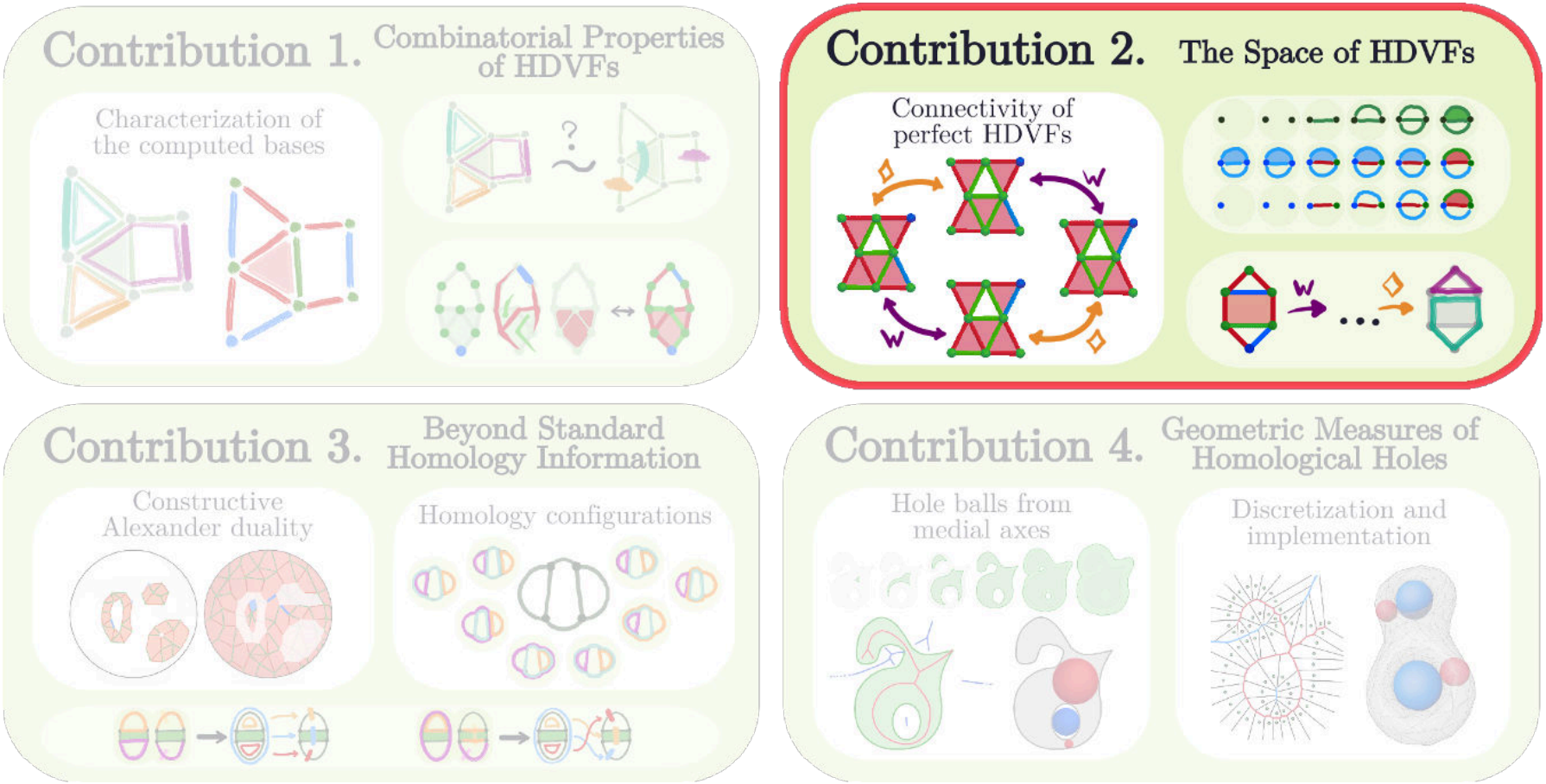
The following bases are explicit:

- Discrete Morse theory bases.
- "Standard" persistent homology bases[1] [2].
- Bases computed by (modified) SNF[3].
- Bases computed by **tri-partitions**[4].



[1] T.Pham et al. *Computing representatives of persistent homology generators with a double twist* (2024)
[2] M.Čufar et al. *Fast computation of persistent homology representatives* (2023)
[3] S.Peltier et al. *Computation of homology groups and generators* (2006)
[4] H.Edelsbrunner and K.Ölsböck. *Tri-partitions and Bases of an Ordered Complex* (2020)

2. The Space of HDVFs and Filtrations

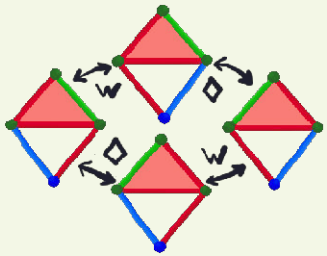


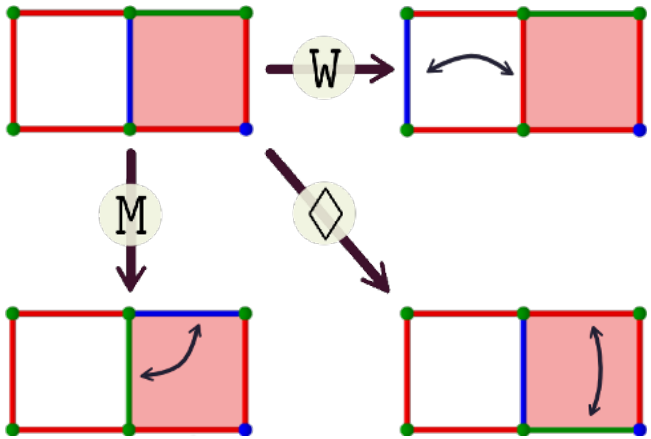
- **Yann-Situ Gazull**, Aldo Gonzalez-Lorenzo and Alexandra Bac. "On the Graph of Homology Computations and its Connectedness", (2025) **preprint**.

Investigating the graph of HDVF

Definition. HDVF graph

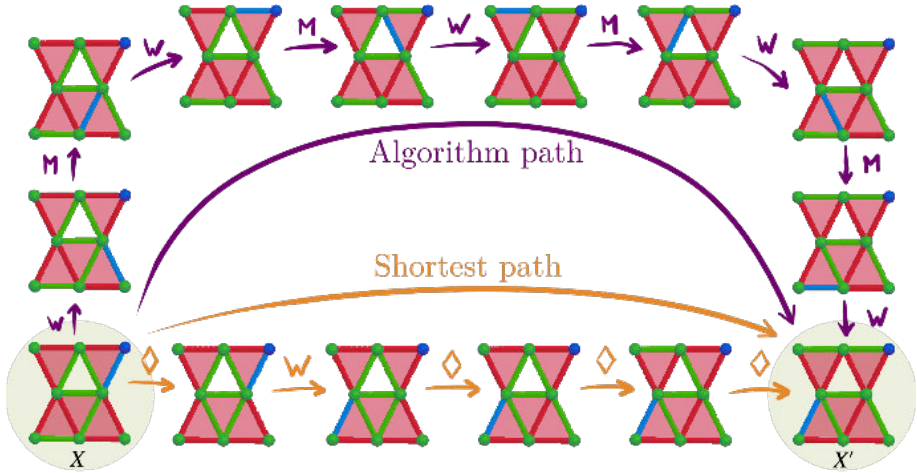
Vertices are **HDVFs**, edges are **swapping operations** (M, W, ◊).





Theorem. (Y-S.Gazull et al. 2025)

The subgraph of **perfect** HDVFs is **connected**.



M1 internship of Menjatiana "Danih" Andrianabinitsoa.

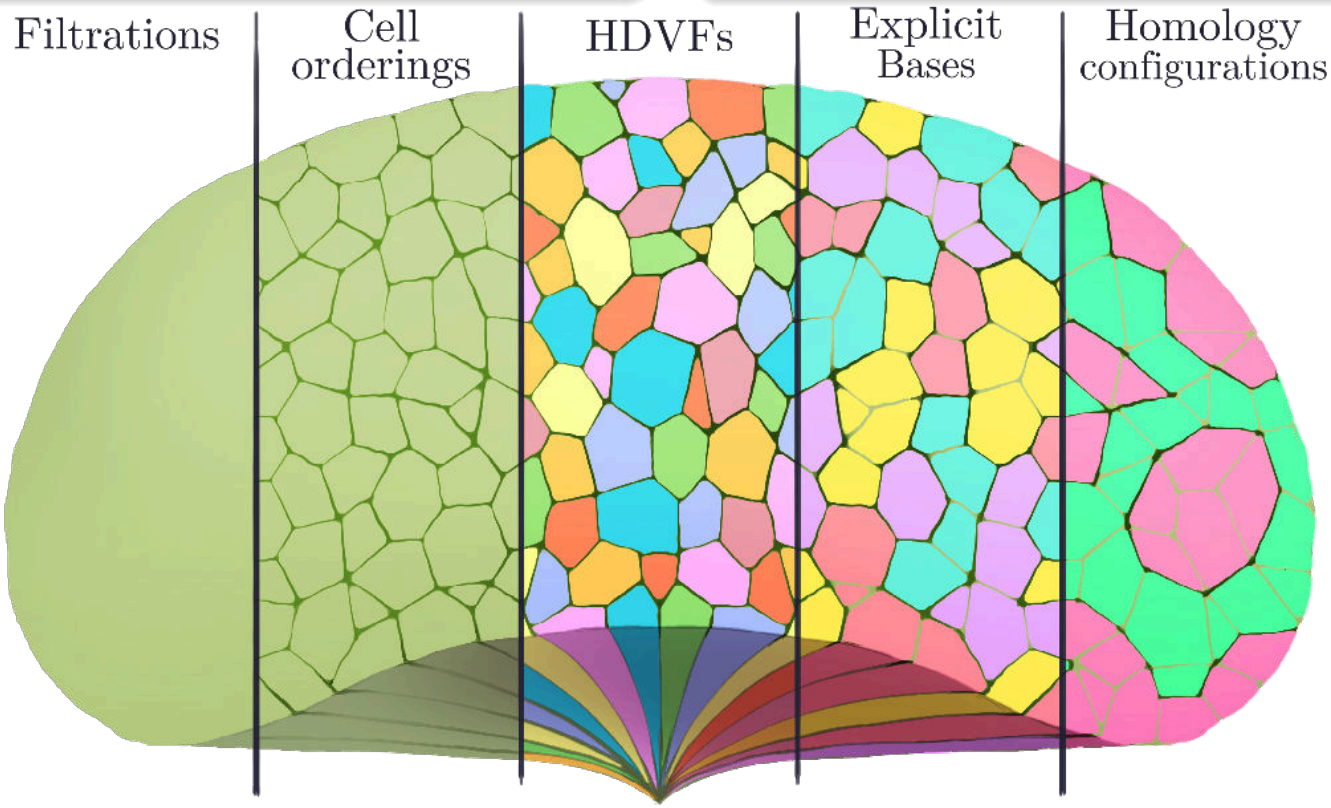
Hierarchies on the filtration space

Proposition. (A.Gonzalez-Lorenzo 2017)

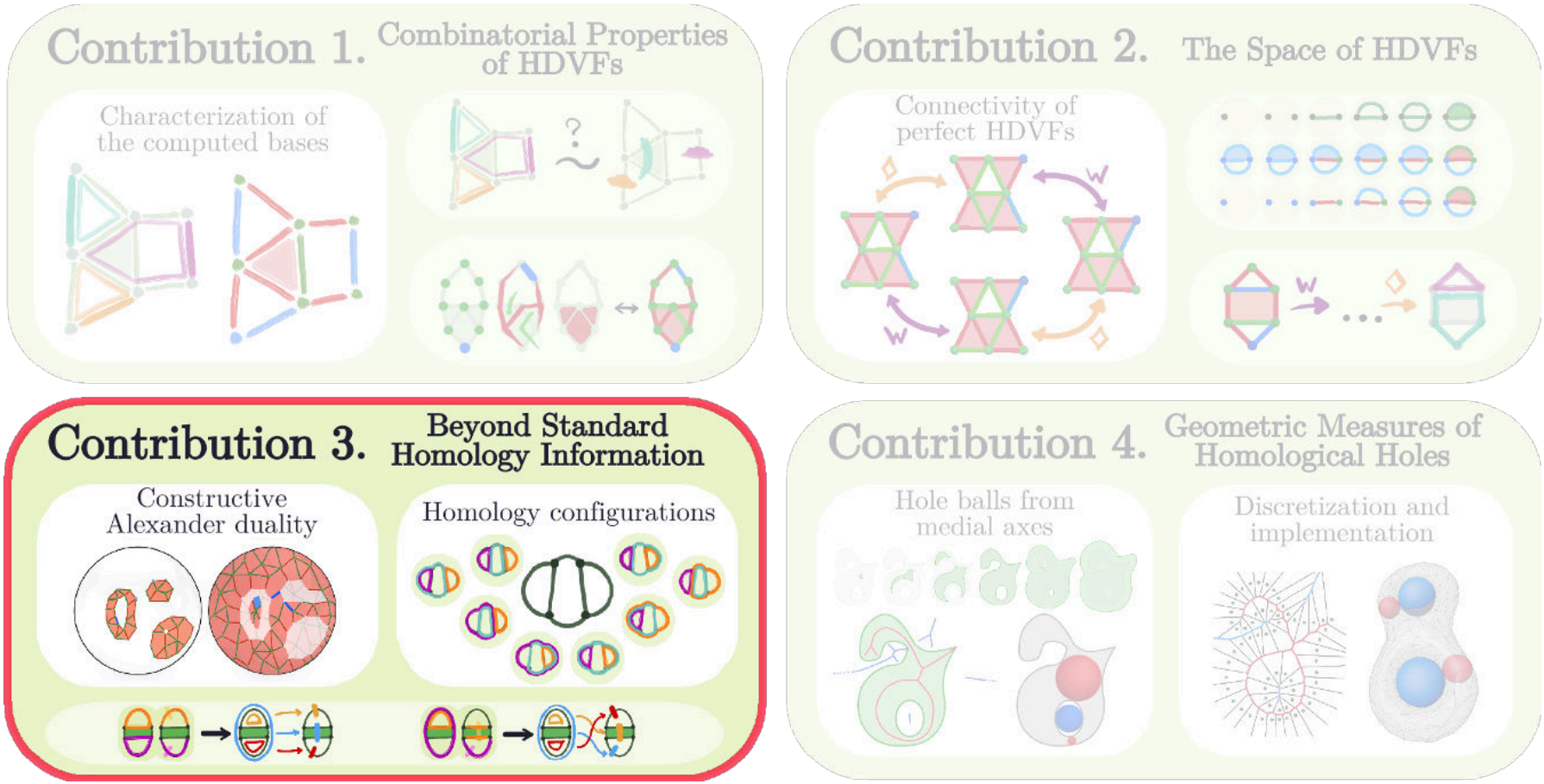
A filtration **induces a unique perfect HDVF**.

Proposition. (Y-S.Gazull)

A perfect HDVF is **induced by at least one filtration**.



3. Beyond Standard Homology Information



- Aldo Gonzalez-Lorenzo, Alexandra Bac and **Yann-Situ Gazull**. "A constructive Approach of Alexander duality". In: *Journal of Applied and Computational Topology 9.2* (Dec. 2024).

A constructive Alexander duality

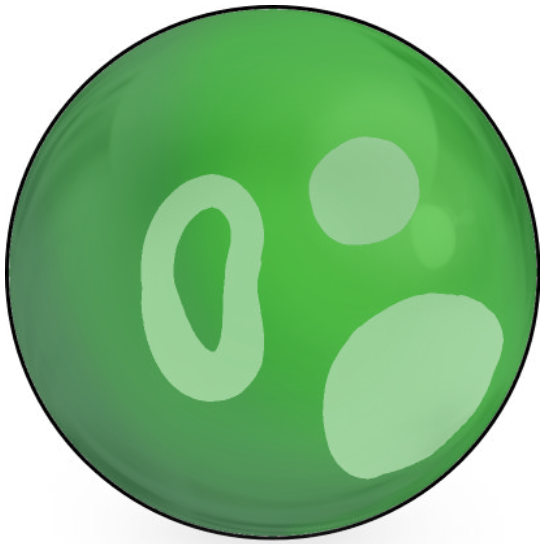
Theorem. Alexander duality (J.W.Alexander 1915 - P.Alexandroff 1927 - L.Pontryagin 1934)

The q-holes of a shape correspond to $(n-q-1)$-holes of **its complementary** in the n-**sphere**.

$$\forall q \geq 0 \qquad \tilde{H}_q(K) \approx \tilde{H}^{n-q-1}(\mathbb{S}^n \backslash K)$$



0-**holes**: (1)+2
1-**holes**: 1



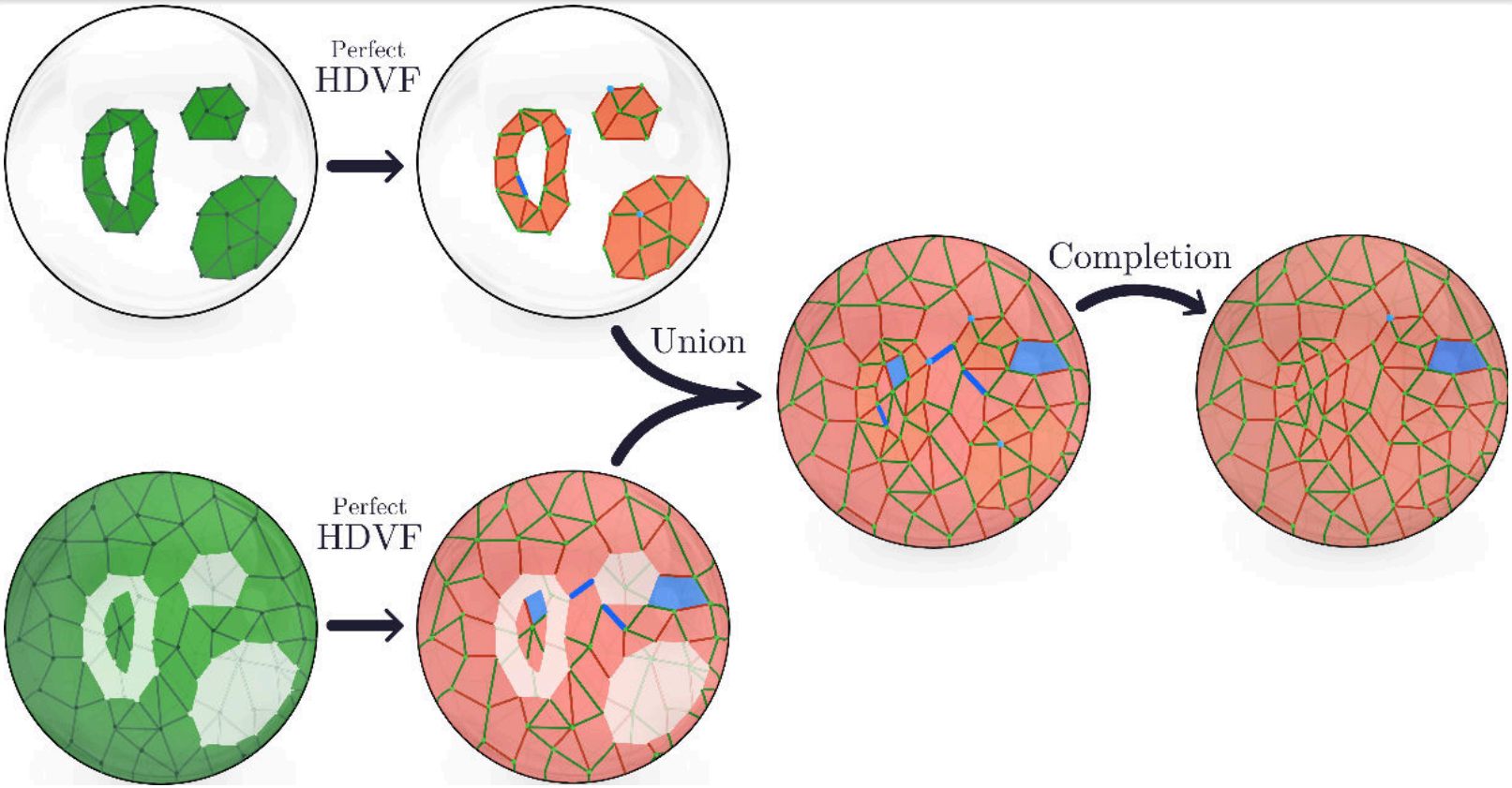
0-**holes**: (1)+1
1-**holes**: 2

A constructive Alexander duality

Theorem. Combinatorial Alexander duality (A.Gonzalez-Lorenzo et al. 2024)

$$\forall q \geq 0 \qquad \tilde{H}_q(K) \approx \tilde{H}^{n-q-1}\left((S \backslash K)^*\right)$$

And there is a HDVF-based **constructive** method to exhibit such an isomorphism.



A constructive Alexander duality

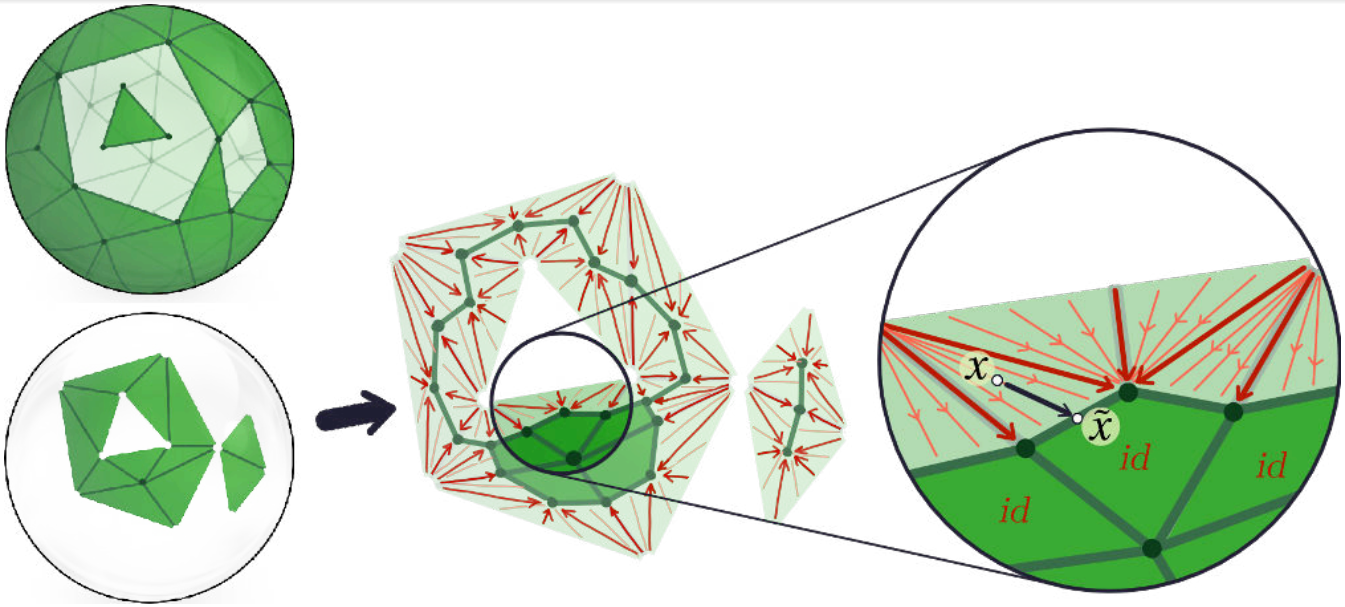
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And there is a HDVF-based **constructive** method to exhibit such an isomorphism.

Theorem. Recovering standard Alexander duality (A.Gonzalez-Lorenzo et al. 2024)

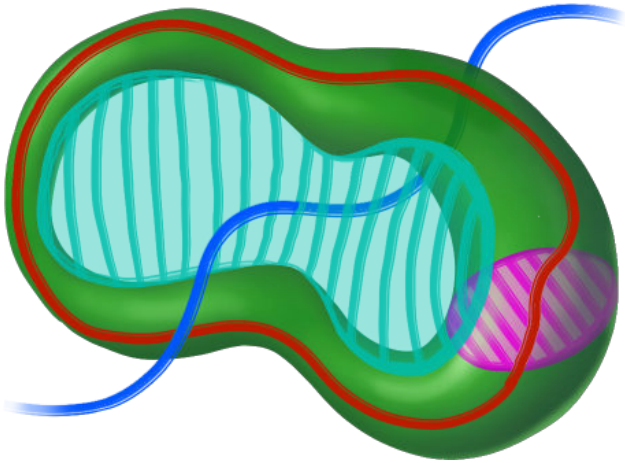
$$\forall q \geq 0 \qquad \tilde{H}_q(|K|) \approx \tilde{H}^{n-q-1}(|S| \backslash |K|)$$

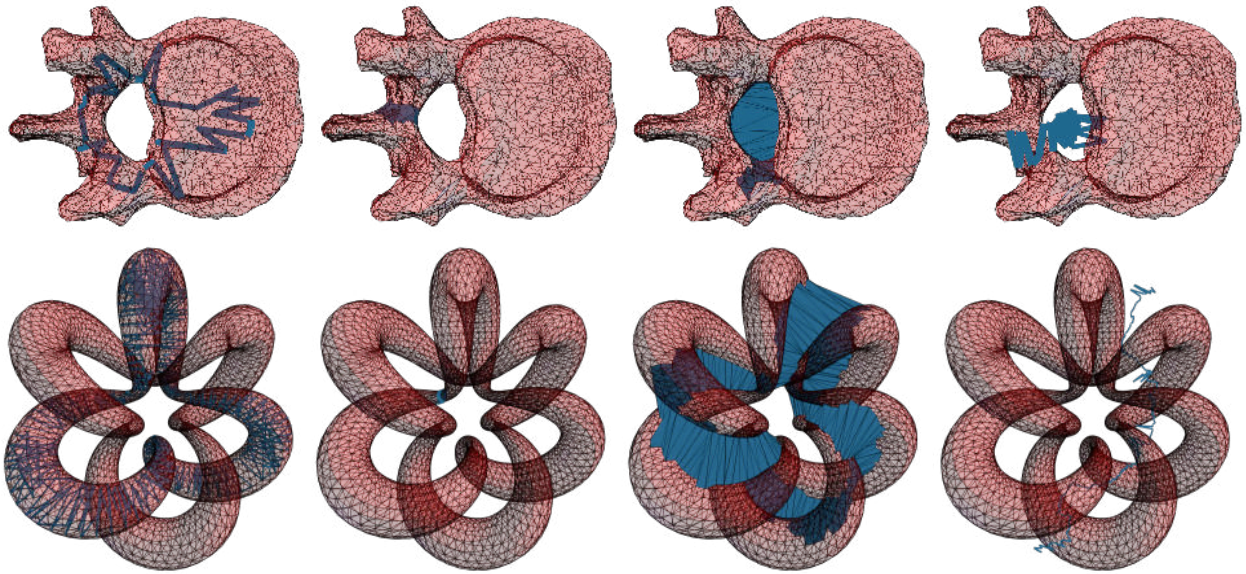


A constructive Alexander duality

Homology quartet

$$\begin{array}{ccc} H_q(K) & \text{———} & H^q(K) \\ | & & | \\ H^{n-q-1}(S\backslash K) & \text{———} & H_{n-q-1}(S\backslash K) \end{array}$$

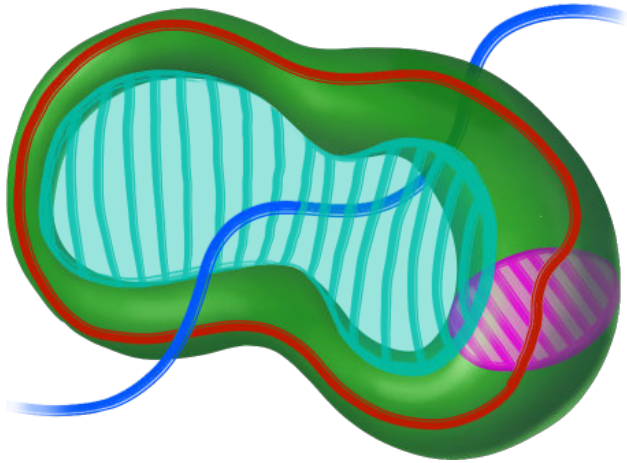




A constructive Alexander duality

Homology quartet

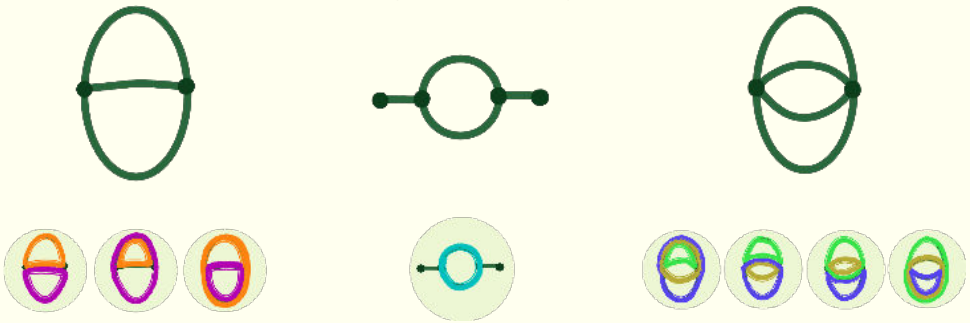
$$\begin{array}{ccc} H_q(K) & \text{———} & H^q(K) \\ | & & | \\ H^{n-q-1}(S\backslash K) & \text{———} & H_{n-q-1}(S\backslash K) \end{array}$$



What isomorphisms are **constructible** with our method?

⇓

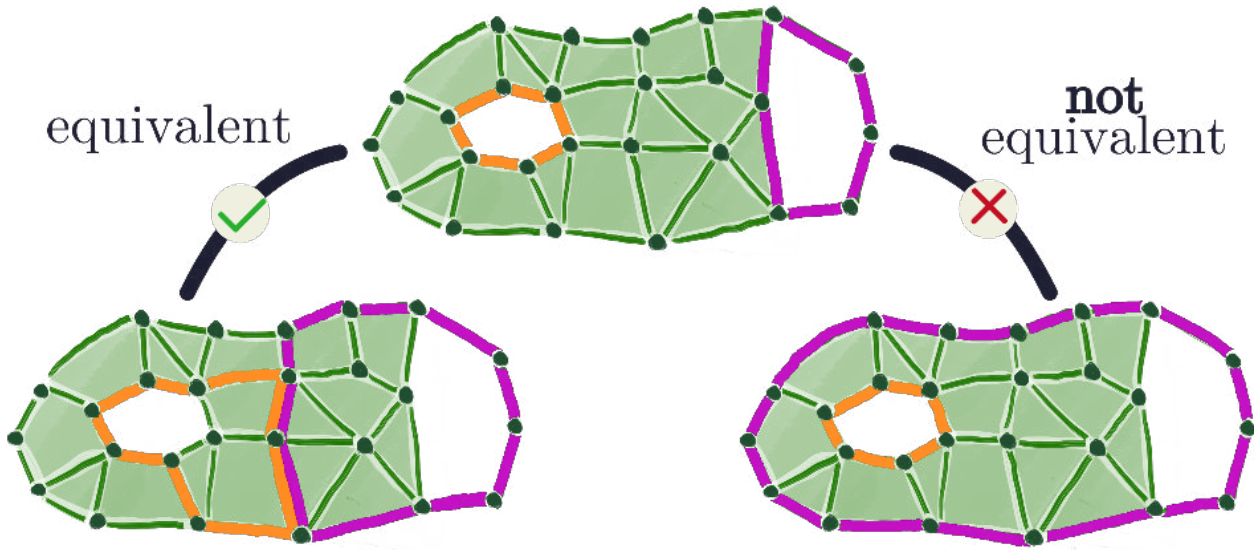
Homology configurations



Homology configurations

Definition.

Two q-homology bases are **equivalent** if they induce the same basis in $H_q(K)$.



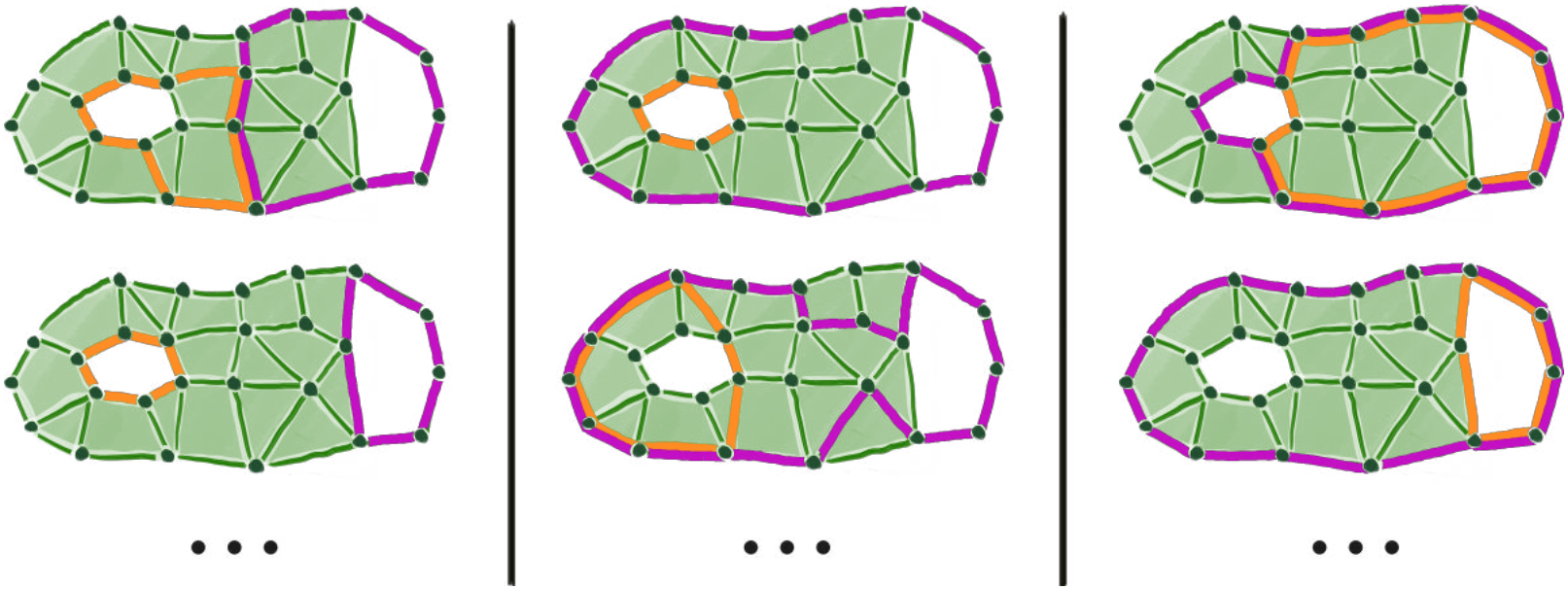
Homology configurations

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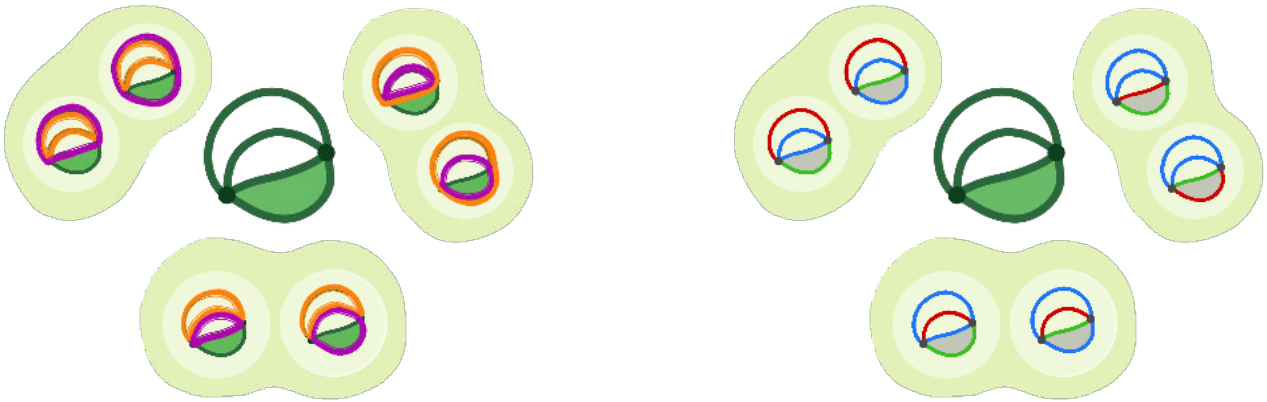
Definition. Homology configurations

The q-**homology configuration** is the set of equivalence classes over **explicit** bases.



Homology configuration definitions and properties

Homology configurations can be also defined for **cohomology**, and as a classification of **perfect HDVFs**, **tri-partitions** or **filtrations**.



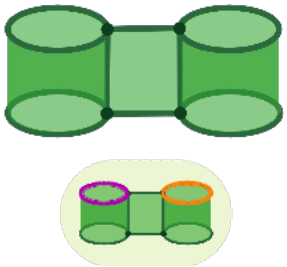
Lemma. (Y-S.Gazull)

All definitions coincide, in particular **homology configurations** coincide with **cohomology configurations**.

A combinatorial invariant

Are homology configurations preserved under subdivision?

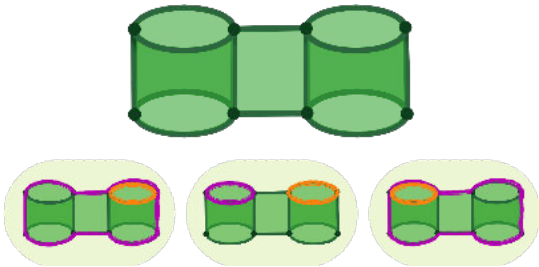
2-complex



1-configurations

subdivision $\longrightarrow$

2-complex

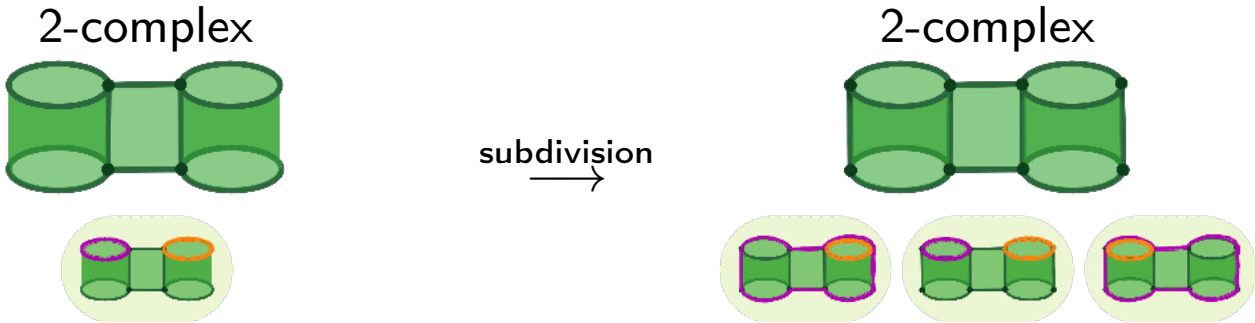


1-configurations

In general **no**...

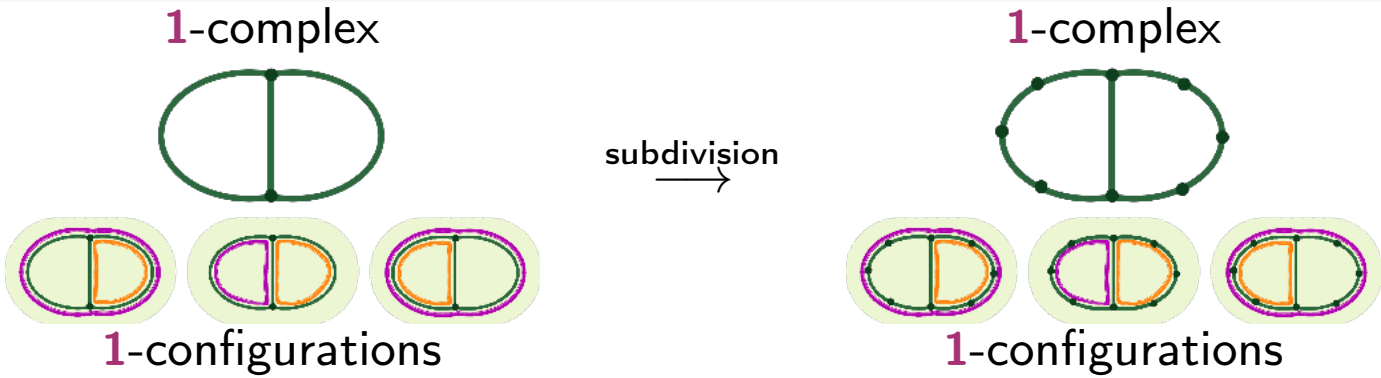
A combinatorial invariant

Are homology configurations preserved under subdivision?

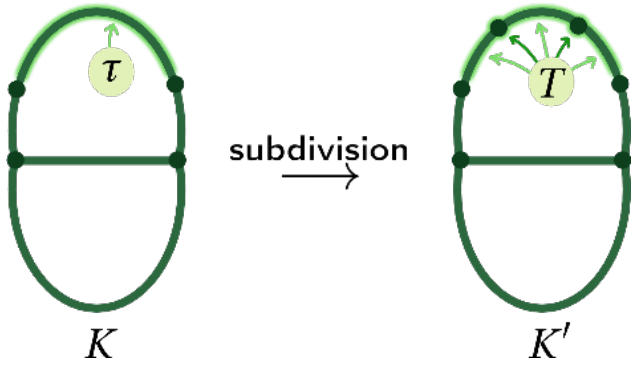


Theorem. (Y-S.Gazull)

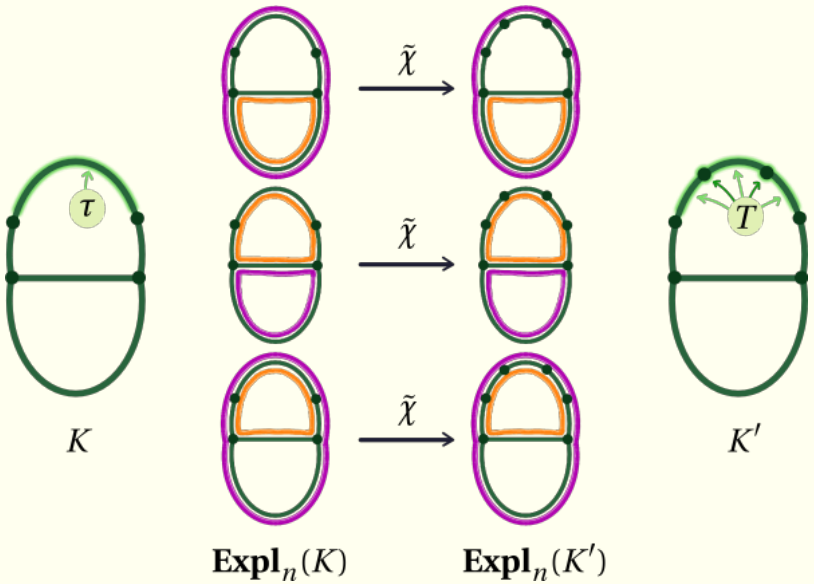
Homology configurations of **maximal dimension** are preserved under **subdivision**.



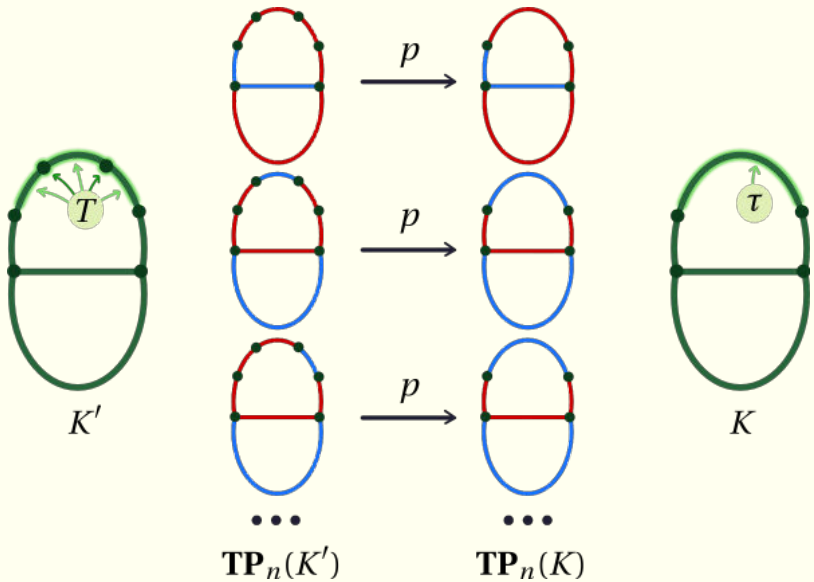
A Combinatorial invariant



Proof. K has fewer configurations than K'



Proof. K' has fewer configurations than K



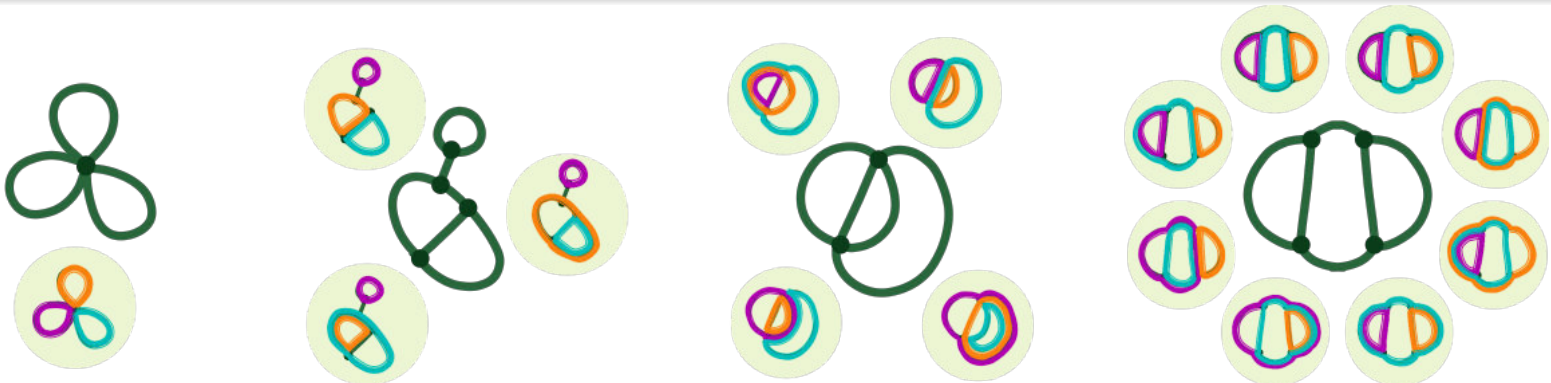
A Topological invariant

Theorem. Hauptvermutung (E.M.Brown 1969)

Homeomorphic simplicial n-complexes admit a common subdivision ($n \leq 3$).

Corollary. (Y-S.Gazull)

The set of homology configurations of **maximal dimension** is a **topological invariant** for complexes of dimension 3 or lower.



homeomorphic $\Longrightarrow$ homotopy equivalent $\Longrightarrow$ homologous

homologous + same number of configurations

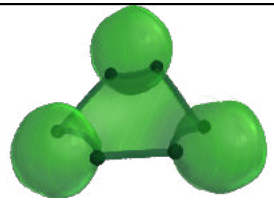
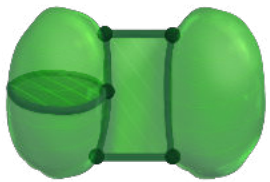
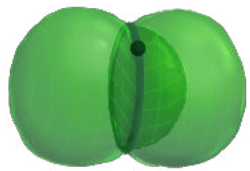
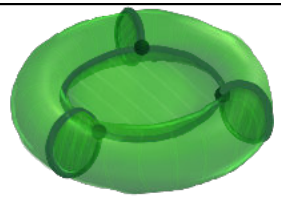
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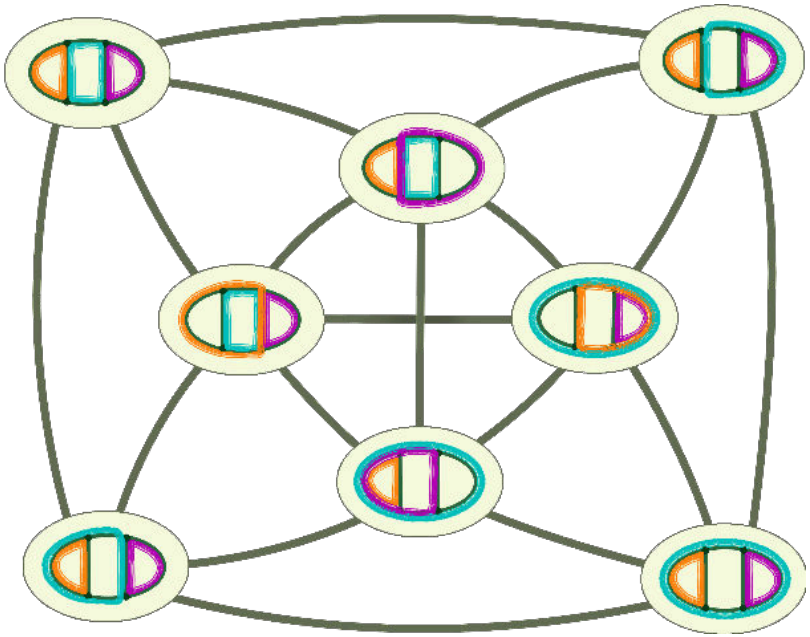
The set of homology configurations of **maximal dimension** is a **topological invariant** for complexes of dimension 3 or lower.

	number of n-configurations (with $n = \dim K$)				
	1	3	4	8	16
$n = 1$					
$n = 2$					

Computing homology configurations

Proposition. (Y-S.Gazull)

The set of homology configurations of maximal dimension has a **(connected) graph structure**, whose edges correspond to **HDVF swapping operations**.



The number of q-configurations is bounded by

$$\frac{1}{\beta_q!}\prod_{i=0}^{\beta_q-1}\left(2^{\beta_q}-2^i\right)$$

Homology morphisms constructible by HDVFs

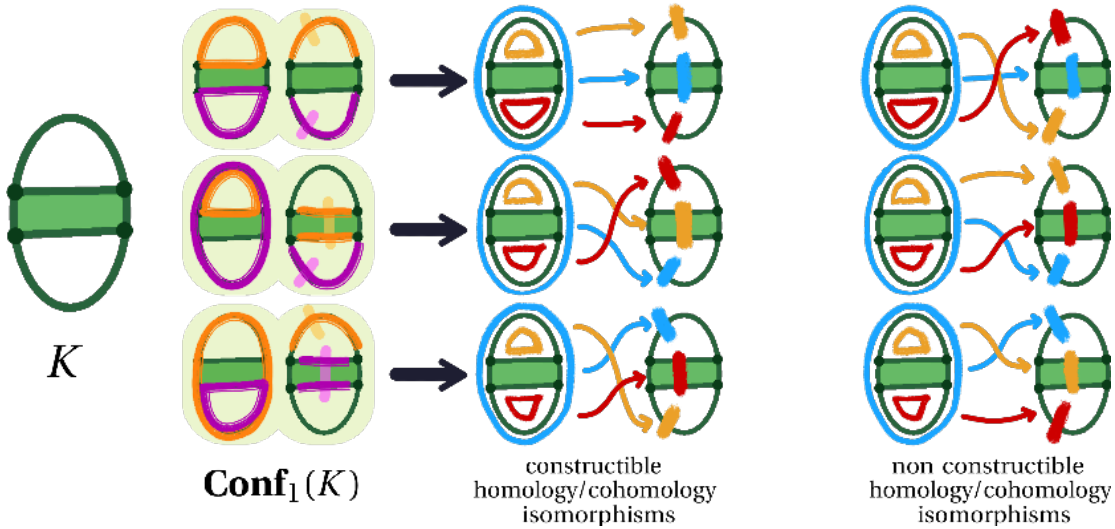
Proposition. (Y-S.Gazull)

Two **pairs of HDVFs** in the same homology configurations (pairwise) induce the **same combinatorial Alexander duality isomorphisms**.

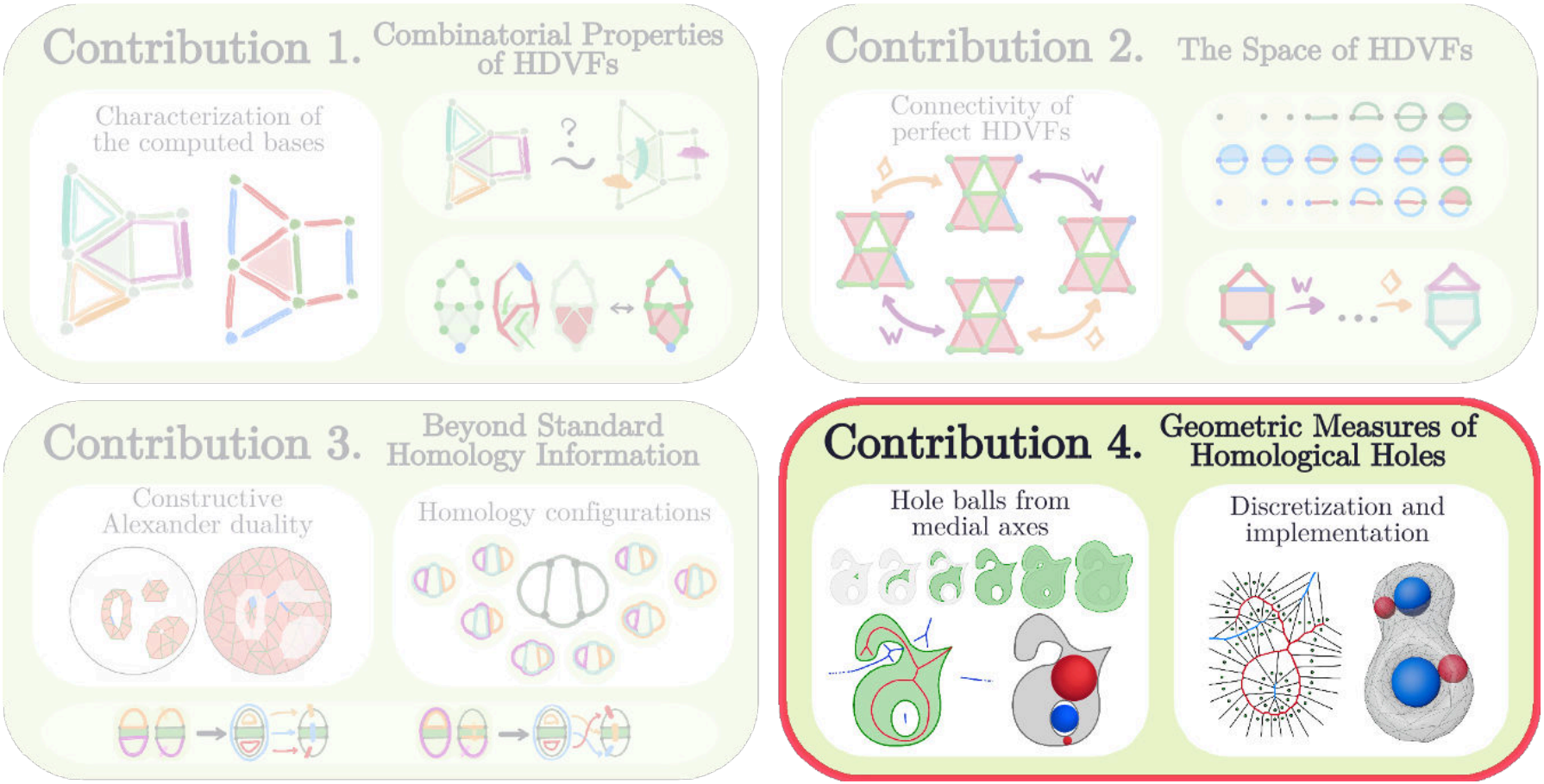
Proposition. (Y-S.Gazull)

Each configuration produces a single **homology-cohomology isomorphism**.

$$H_q(K) \to H^q(K)$$



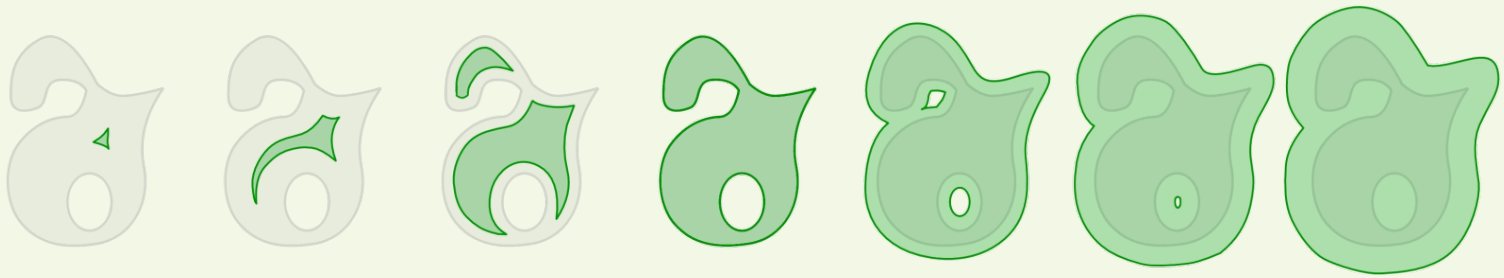
4. Geometric Measures for Homological Holes



- **Yann-Situ Gazull**, Alexandra Bac and Aldo Gonzalez-Lorenzo. "Computing Geometrical Measures of Topological Holes". In: SPM23 special issue, *Computer-Aided Design 163* (2023).

Hole Balls

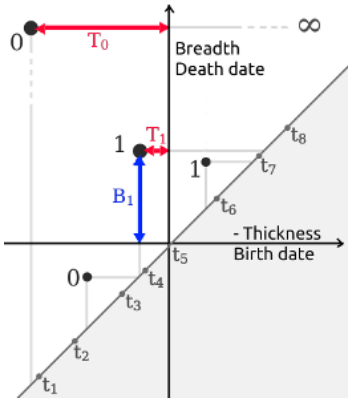
Definition. Signed distance field filtration (*sdf*-filtration)

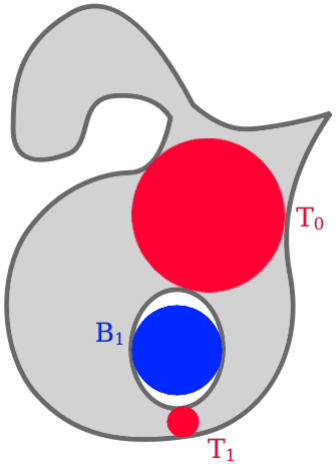


⇓

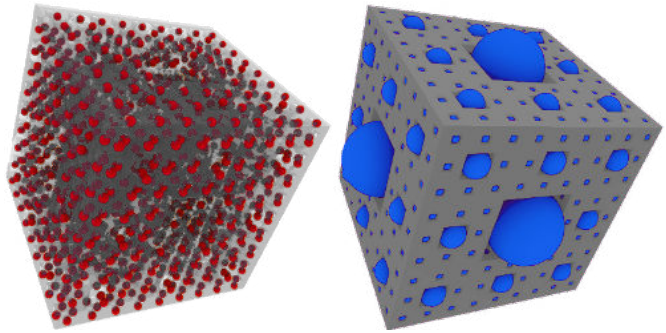
persistent homology

⇓





Thickness and breadth balls



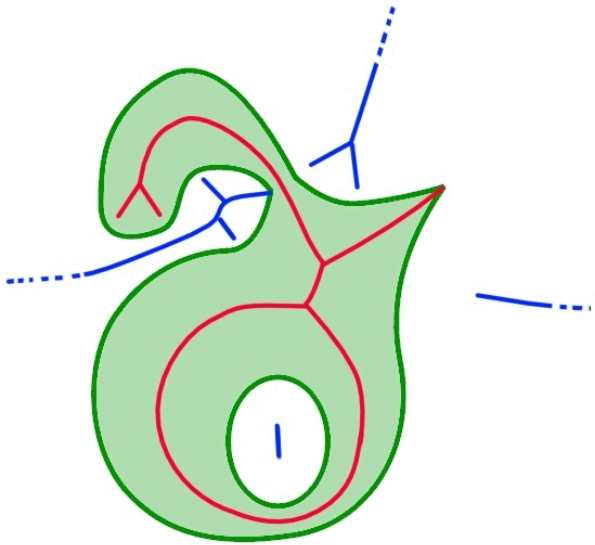
A.Gonzalez-Lorenzo et al.: *Two Measures for the Homology Groups of Binary Volumes (2016)*

The medial axis

Definition. Medial axis

The set of points that have **two or more closest boundary points**.





inner and outer medial axes

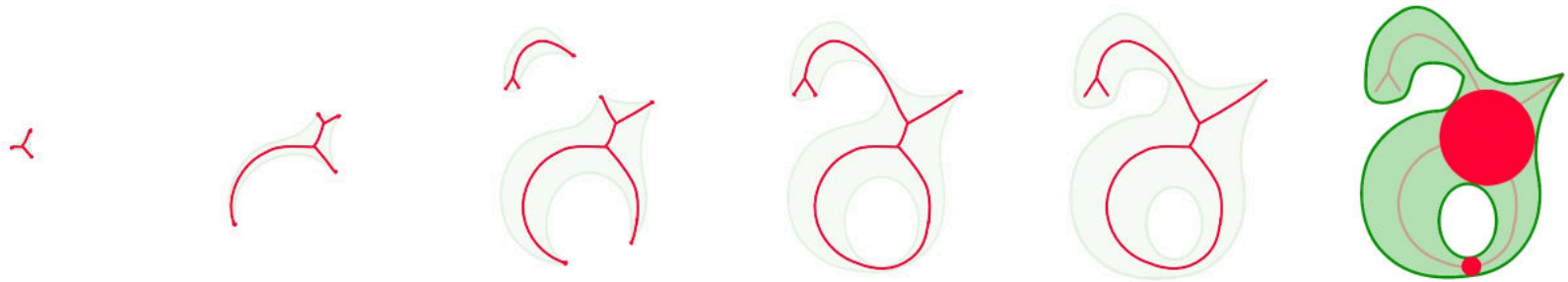
Theorem. (A.Lieutier 2003)

Every bounded open set is homotopy equivalent to its inner medial axis.

Theoretical methods

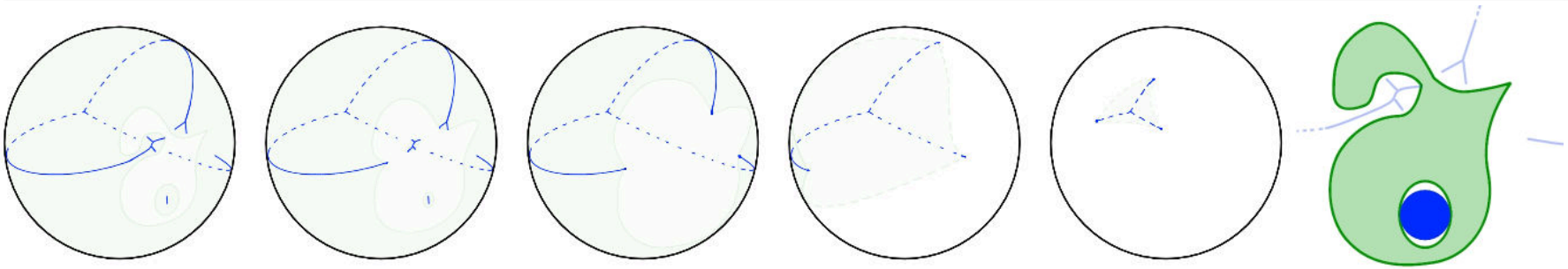
Proposition. (Y-S.Gazull et al. 2023)

Sdf-persistence for $t \leq 0$ is equivalent to sdf-persistence restricted to the **inner medial axis**.



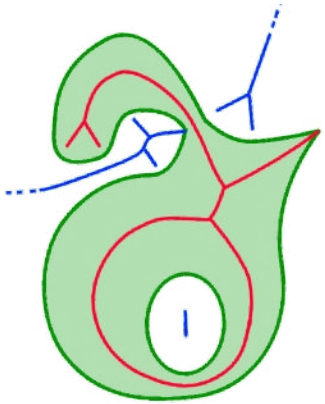
Proposition. (Y-S.Gazull et al. 2023)

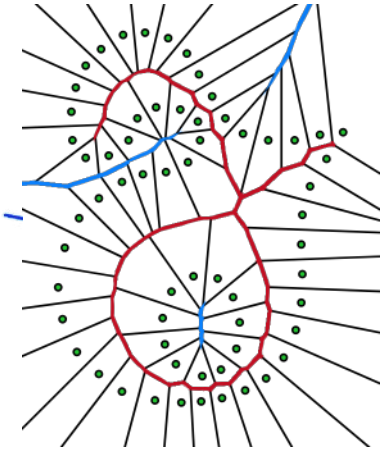
Sdf-persistence for $t \geq 0$ can be recovered from sdf-persistence restricted to the **outer medial axis** (using Alexander duality).



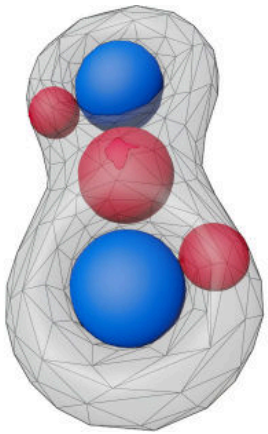
Practical methods and experiments

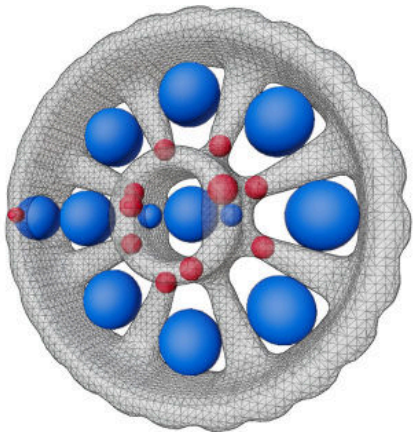
Discretization of the methods using **Voronoi diagram** of a sampling on the boundary of the shape.

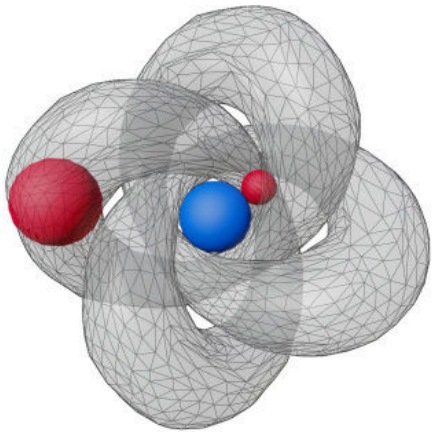


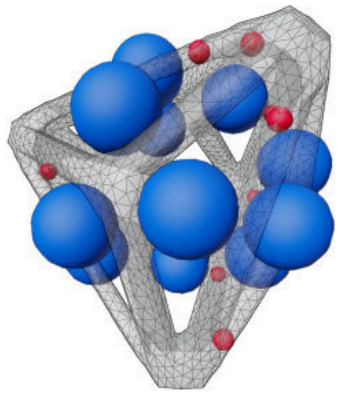


Implementation in **C++**, using **CGAL** and **Phat**.



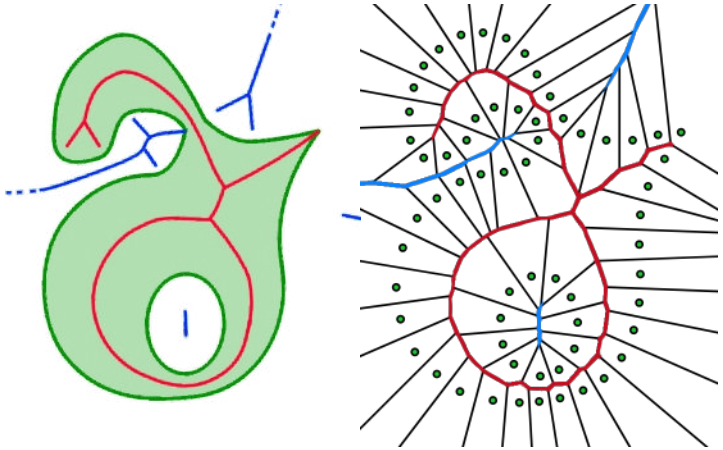




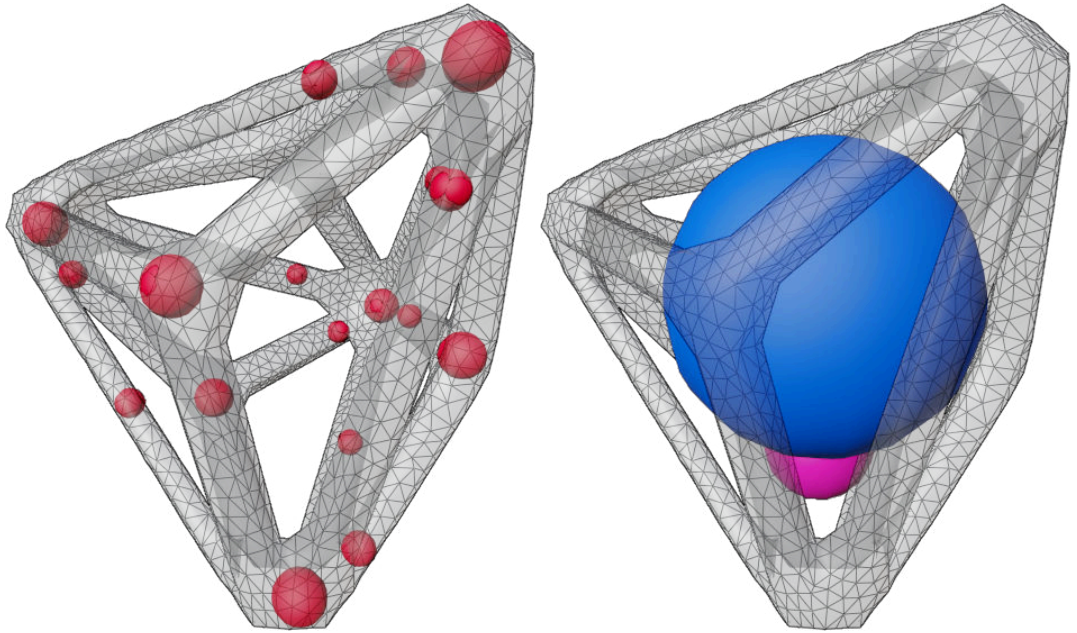


Practical methods and experiments

Discretization of the methods using **Voronoi diagram** of a sampling on the boundary of the shape.

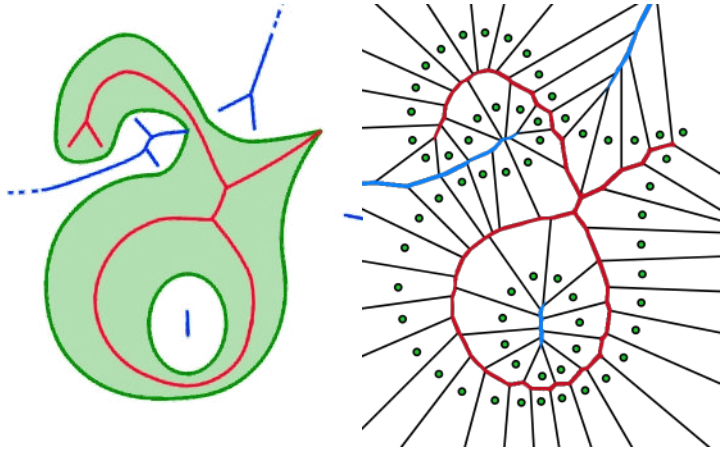


Implementation in **C++**, using **CGAL** and **Phat**.



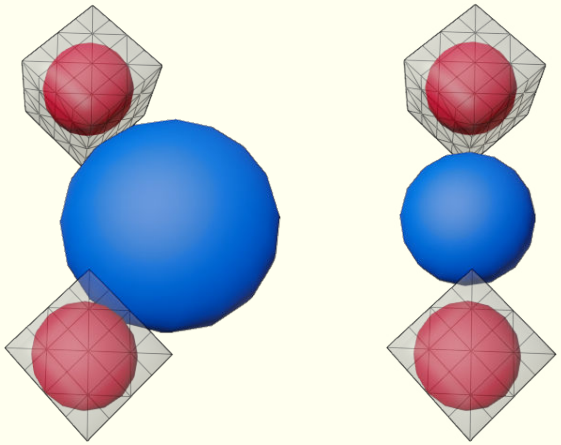
Practical methods and experiments

Discretization of the methods using **Voronoi diagram** of a sampling on the boundary of the shape.

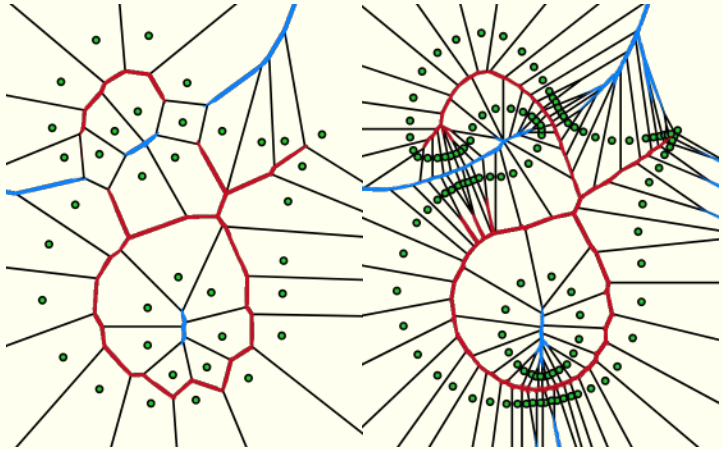


Implementation in **C++**, using **CGAL** and **Phat**.

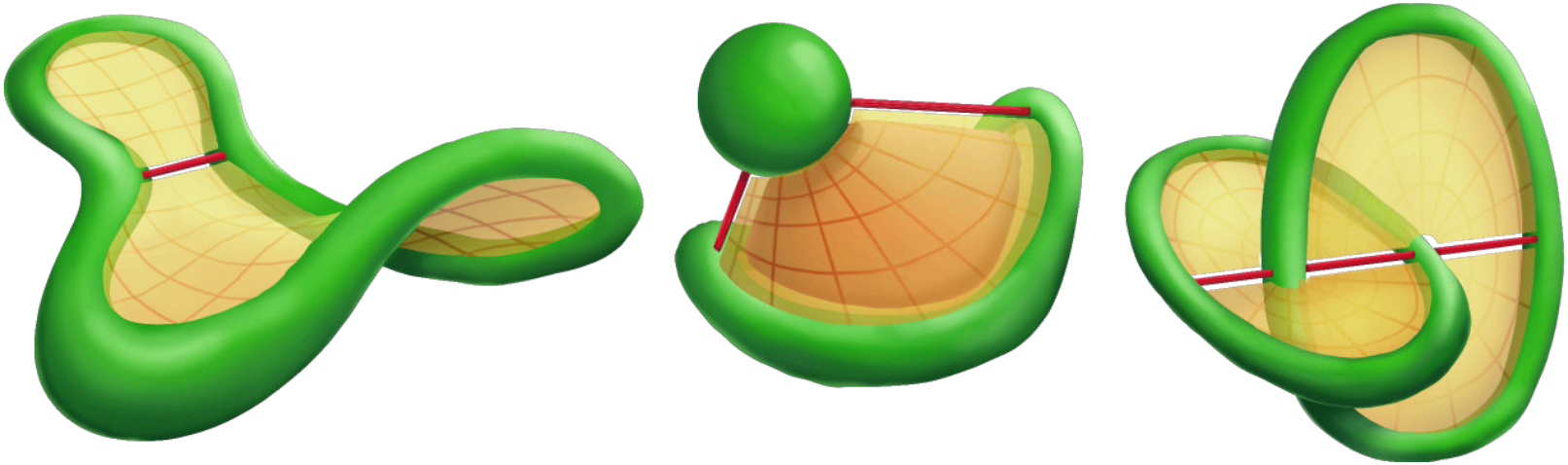
Challenge: catching the ball center

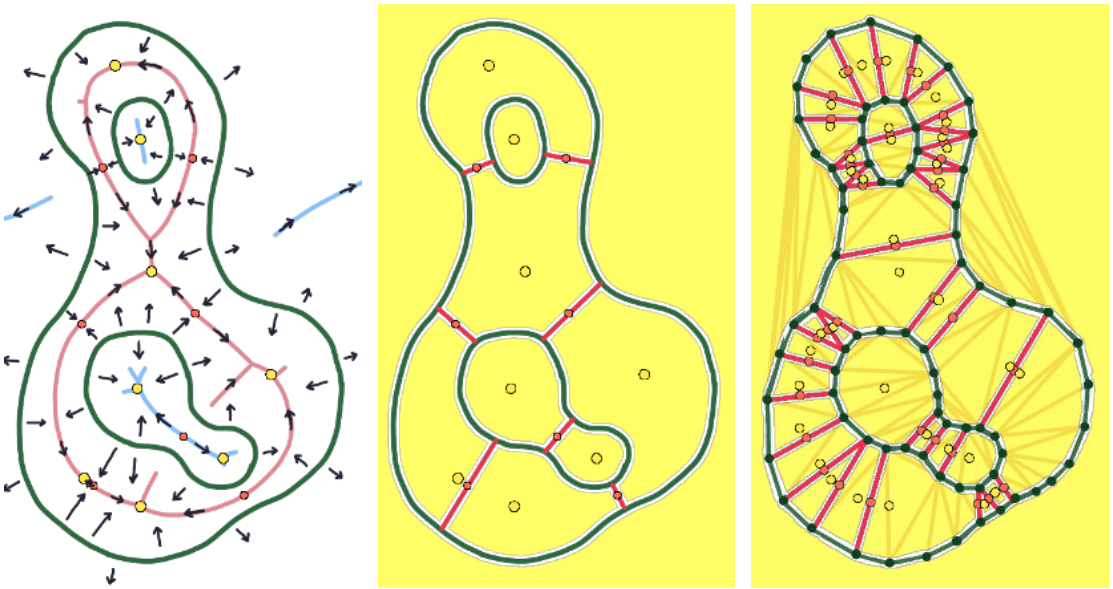


Limitation: sampling density



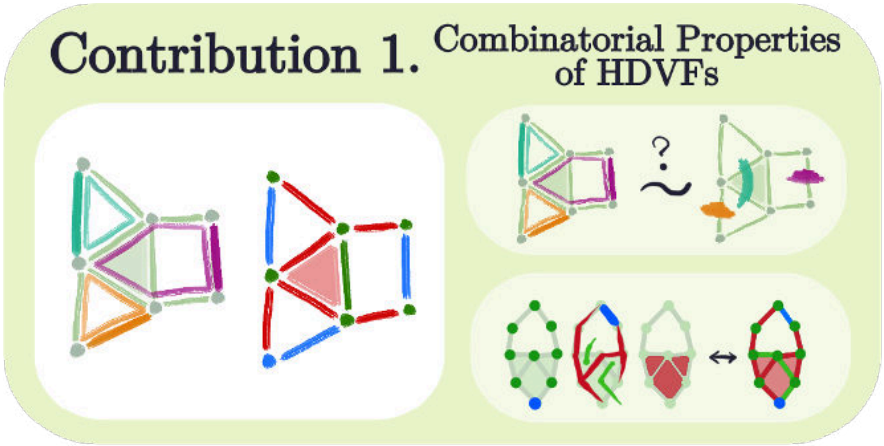
Computing the shape of the holes?

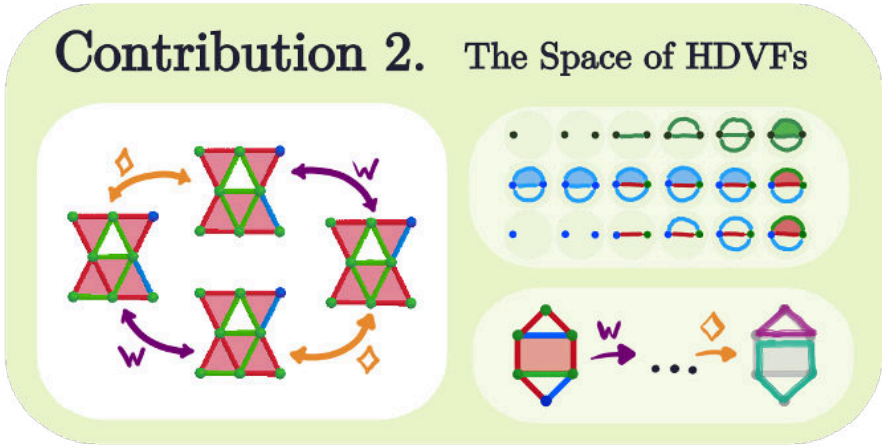


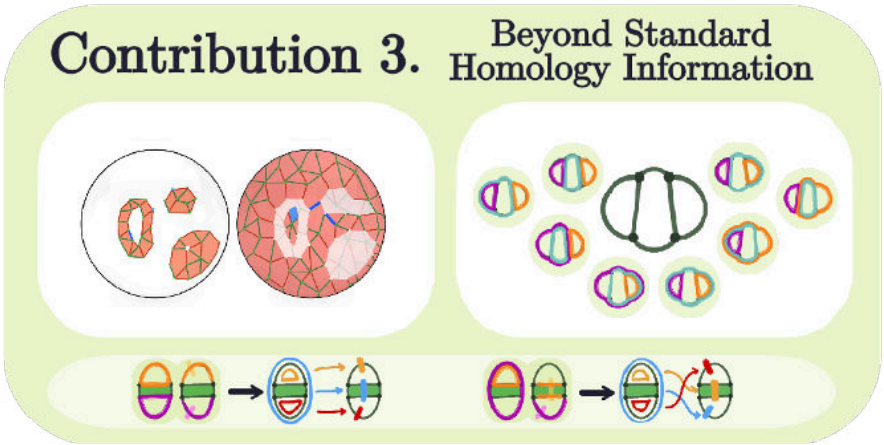


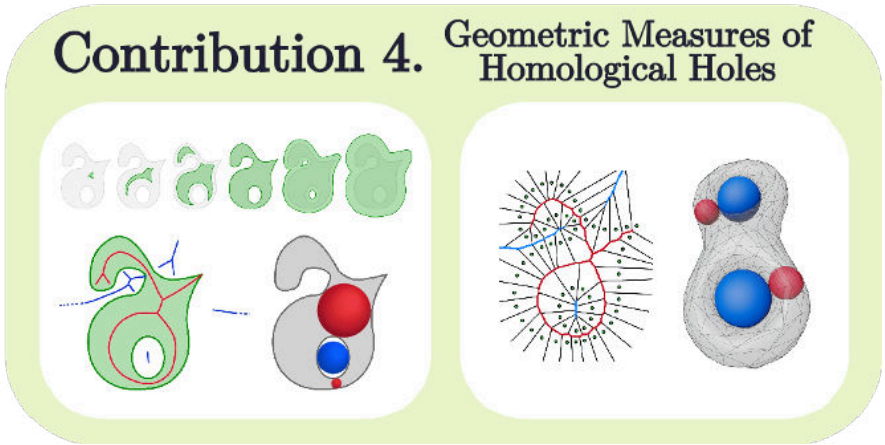
M2 internship of Madhav Cherupilil Sajeev.

Conclusion: Summary of the Contributions









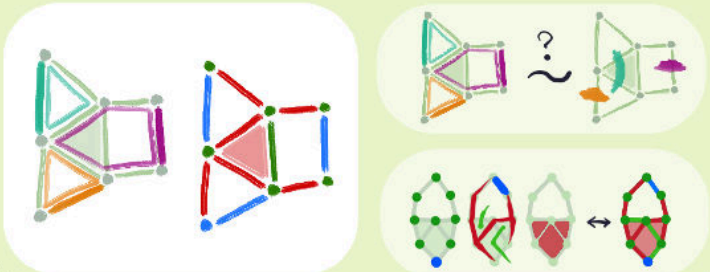
1. **Y-S.Gazull**, A.Bac and A.Gonzalez-Lorenzo. *Computing Geometrical Measures of Topological Holes*. In: SPM23 special issue, *CAD* (2023).
2. A.Gonzalez-Lorenzo, A.Bac and **Y-S.Gazull**. *A constructive Approach of Alexander duality*. In: *JACT* (2024).
3. **Y-S.Gazull**, A.Gonzalez-Lorenzo and A.Bac. *Characterization of the Computed Homology and Cohomology bases*. In *the proceedings of DGMM* (2025).
4. **Y-S.Gazull**, A.Gonzalez-Lorenzo and A.Bac. *On the Graph of Homology Computations and its Connectedness*, (2025) preprint.
5. **Y-S.Gazull**. *Stability and extension of steady and ranging persistence*. In *JACT* (2025).
6. **Y-S.Gazull**, A.Gonzalez-Lorenzo and A.Bac. *Characterization of Homology and Cohomology bases Computed by Standard Methods*. In: DGMM25 special issue, *JMIV* (2026), to be published.

A combinatorial framework

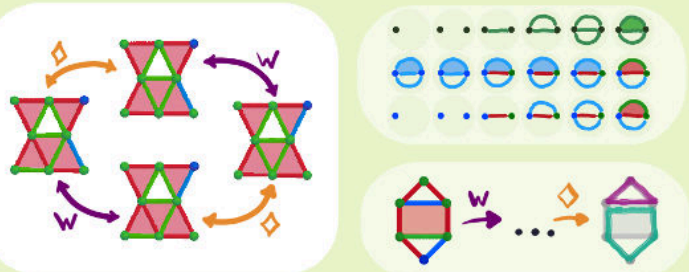
Global points of view

Representations of holes

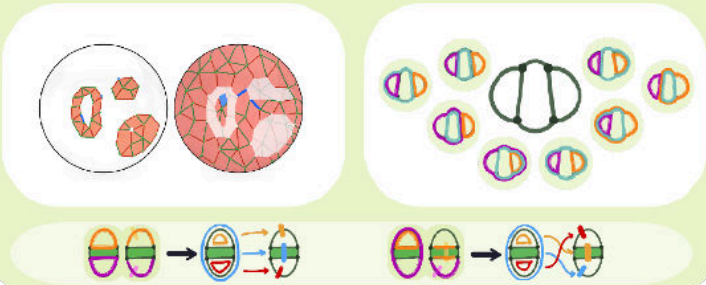
Contribution 1. Combinatorial Properties of HDVFs



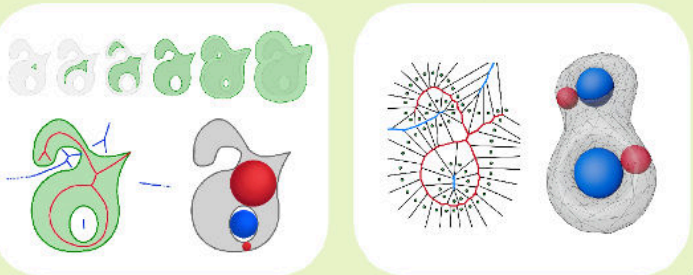
Contribution 2. The Space of HDVFs



Contribution 3. Beyond Standard Homology Information



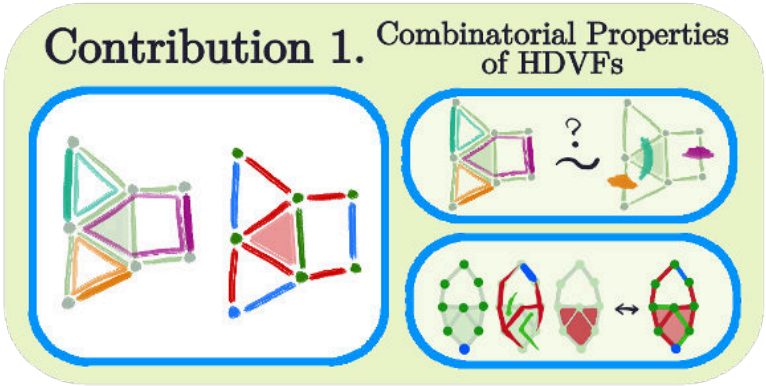
Contribution 4. Geometric Measures of Homological Holes

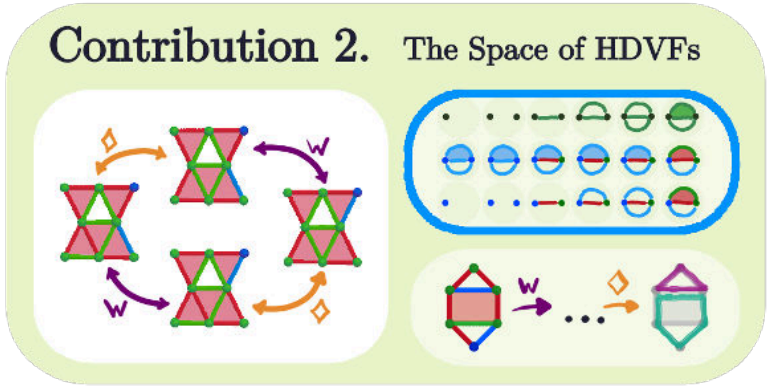


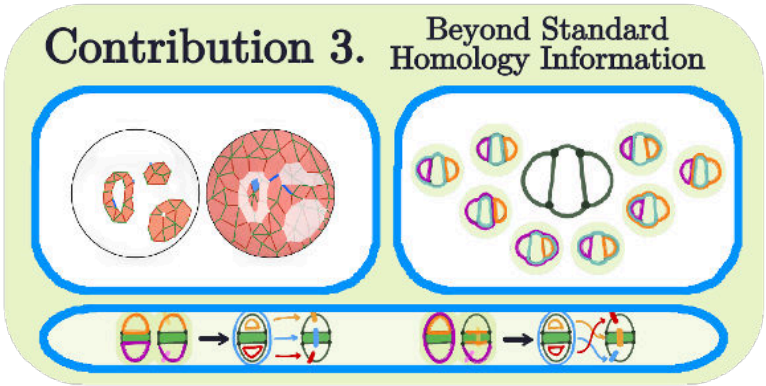
A combinatorial framework

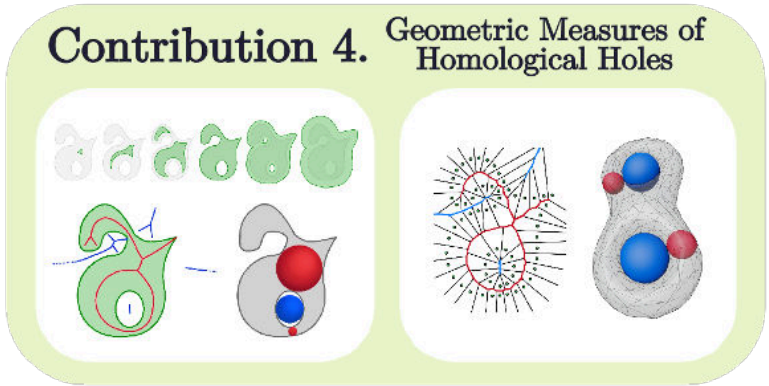
Global points of view

Representations of holes









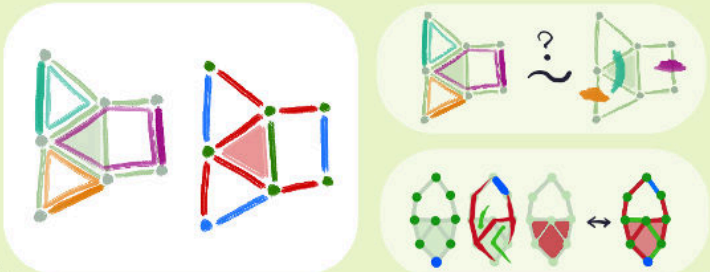
HDVFs: a combinatorial framework to study homology computations.

A combinatorial framework

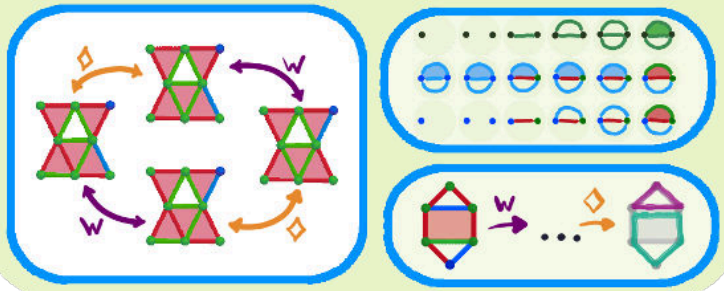
Global points of view

Representations of holes

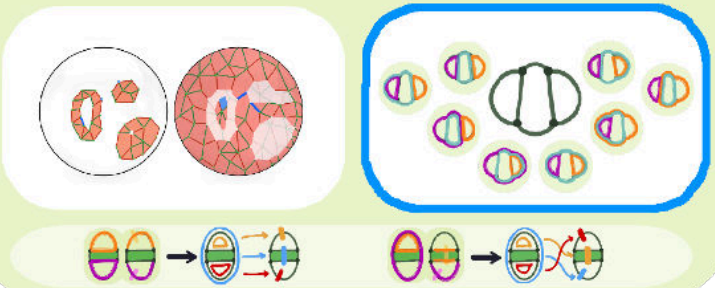
Contribution 1. Combinatorial Properties of HDVFs



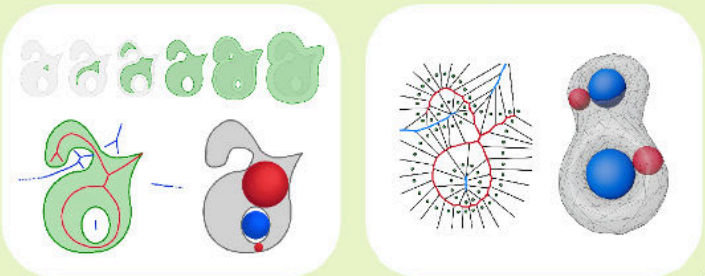
Contribution 2. The Space of HDVFs



Contribution 3. Beyond Standard Homology Information



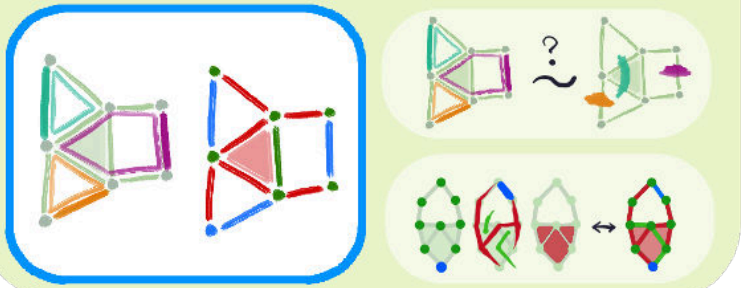
Contribution 4. Geometric Measures of Homological Holes



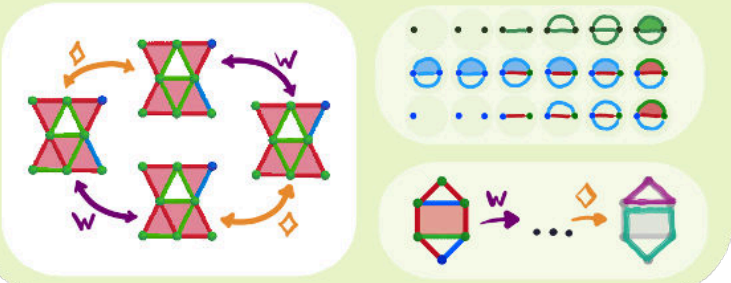
Global points of view on homology computations.

A combinatorial framework | **Global points of view** | **Representations of holes**

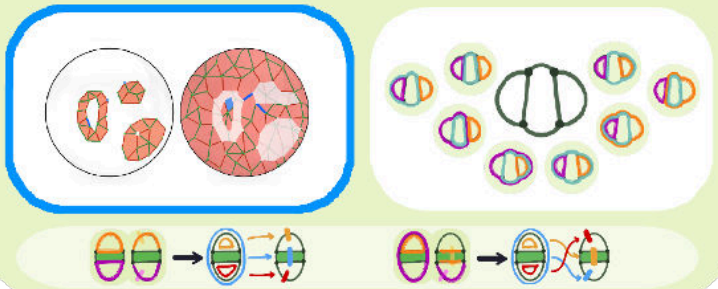
Contribution 1. Combinatorial Properties of HDVFs



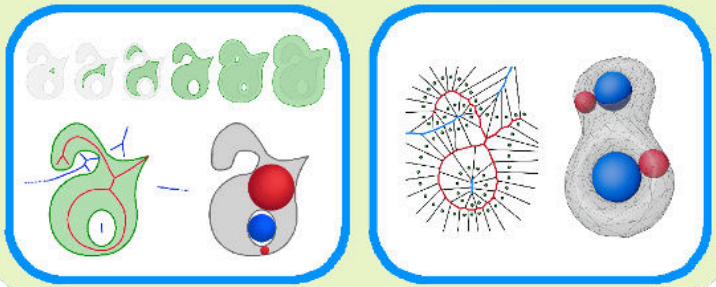
Contribution 2. The Space of HDVFs



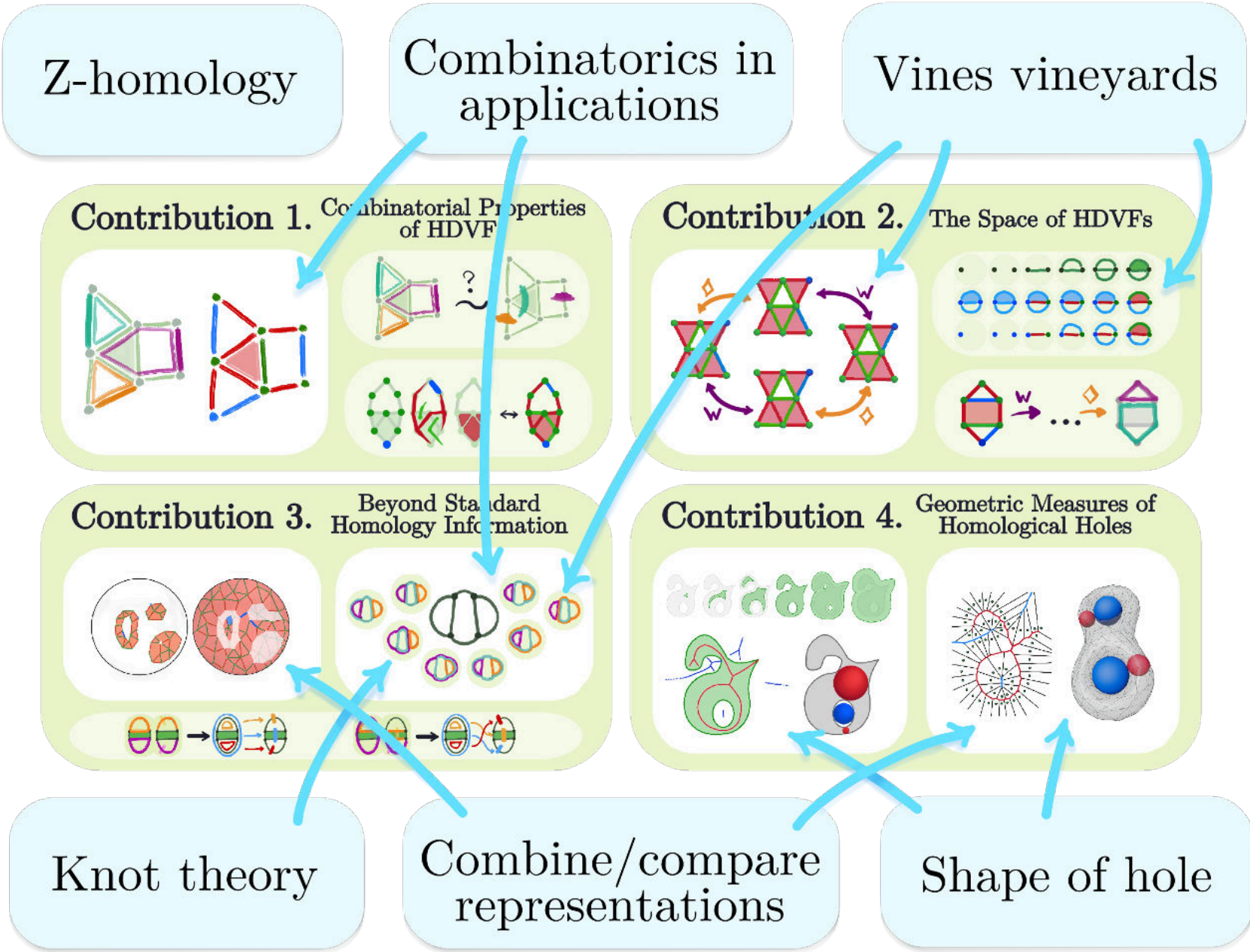
Contribution 3. Beyond Standard Homology Information



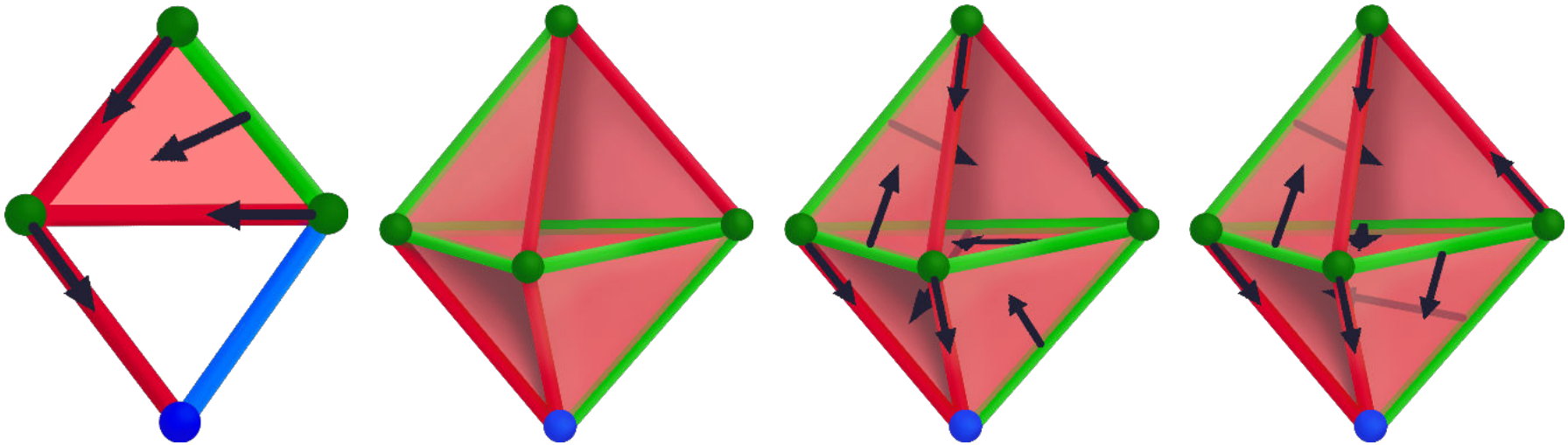
Contribution 4. Geometric Measures of Homological Holes

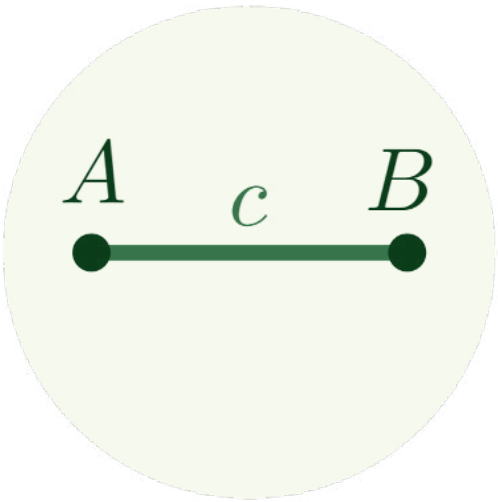


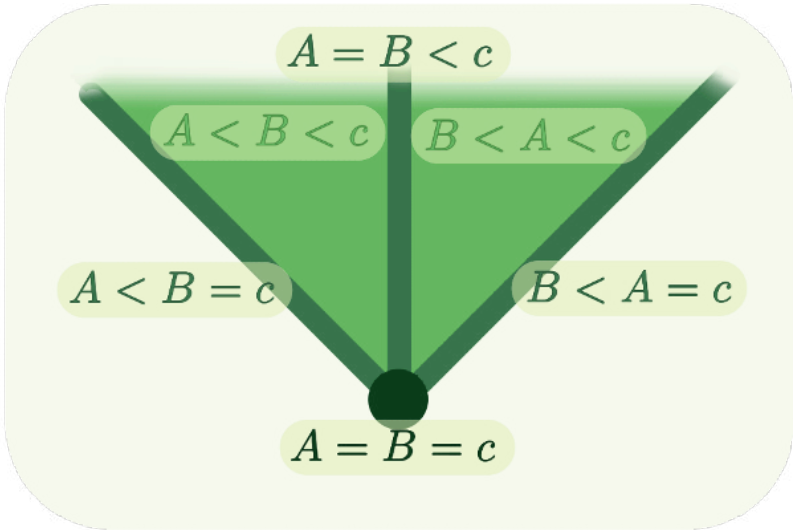
Geometric and combinatorial representations of holes.

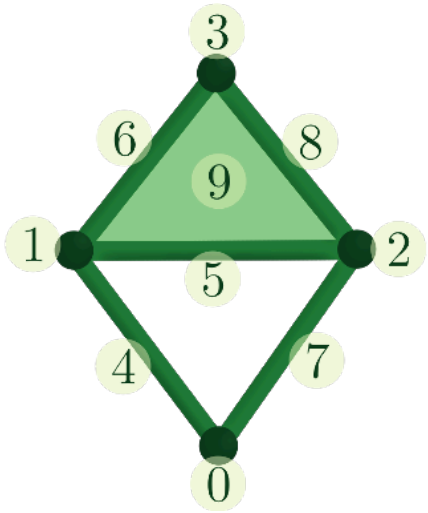

Z-homology
Combinatorics in applications
Vines vineyards
Contribution 1.
Combinatorial Properties of HDVF
Contribution 2.
The Space of HDVFs
Contribution 3.
Beyond Standard Homology Information
Contribution 4.
Geometric Measures of Homological Holes
Knot theory
Combine/compare representations
Shape of hole

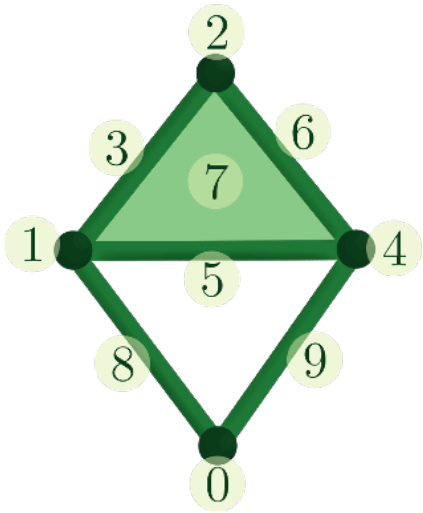
Questions?

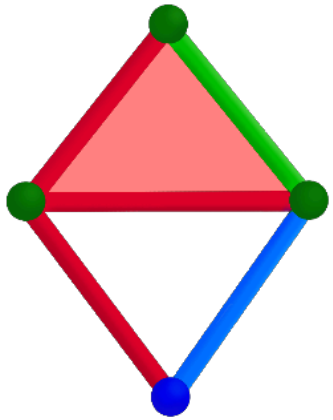



A
c
B

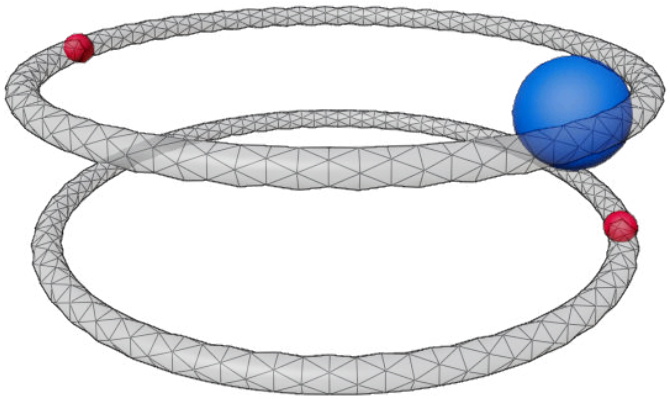

A = B < c
A < B < c
B < A < c
A < B = c
B < A = c
A = B = c

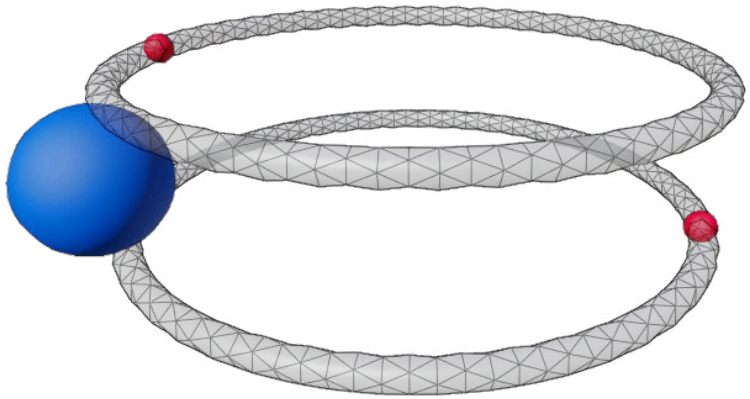

3
6
8
9
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2
5
4
7
0


2
3
6
7
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0



Degeneracy





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